Contrasting Cases – Estimation Study
Fall 2006
Introduction Lesson

Overall Goal: In about 10 minutes, 1) introduce students to the idea of estimation as getting an approximate answer – and compare estimation with getting the exact answer, 2) introduce a less familiar estimation strategy. Note that there is no separate introduction lesson to simplicity, so it is OK if the idea of simplicity comes up during this lesson.

Pass-out Handout with problems and student solutions

Problem 1:

Mrs. Roseman needs to buy some candy for the Halloween trick-or-treaters in her neighborhood. Candy is on sale for $2.98 per bag. Mrs. Roseman wants to buy 9 bags of candy. Before tax, about how much will it cost for Mrs. Roseman to buy the candy?

Mollie and Suzanne are solving this problem. Here is what they did:

<table>
<thead>
<tr>
<th>Mollie’s solution</th>
<th>Suzanne’s solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mollie solves the problem by estimating. She rounds $2.98 to $3.00 and then multiplies by 9 like this:</td>
<td>Suzanne solved this problem by multiplying $2.98 * 9, like this:</td>
</tr>
<tr>
<td>$3.00 * 9 = $27.00</td>
<td>$2.98 * 9</td>
</tr>
<tr>
<td>Mollie gets $27.00 as her answer.</td>
<td>26.82</td>
</tr>
</tbody>
</table>

Have a student read the problem statement

Questions that might be useful to ask students about these solutions:

(1) How did Mollie solve the problem?
(2) How did Suzanne solve the problem?

(2) When is it OK to estimate like Mollie did? When is it not OK to estimate?

(OK to estimate): These examples are ones where it doesn’t matter that you know that exact result, but a rough guess/estimate is good enough. When you want to know about how much money you need to buy some things; when your family is driving a long way to visit friends, figuring out about how long it will take you to get there; when you want to know about how many people came to see your school play (rows and number per row); when you are saving your money to buy something expensive at the store and you want to know about how many week’s allowance it will take to buy it)
(Not OK to estimate: These examples are ones where it does matter that you know exactly how much of something you are working with. When the person at the store tells you how much money you owe; On a math test, when the problem asks you for the exact answer)

(3) Is Suzanne’s was easy to do in your head?
(4) Is Molly’s way <easy to do in your head>?

Try to avoid comparing two different estimates or ways to estimate
Problem 2: Jamie thinks he has spent over $500 on music CDs, but his brother doesn’t believe him. Jamie does a quick count and figures he has about 72 CDs and that each CD cost about $14. Has Jamie spent over $500 on CDs?

<table>
<thead>
<tr>
<th>Frank’s solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>72 * 14</td>
</tr>
<tr>
<td>My estimate is 700.</td>
</tr>
<tr>
<td>I covered up the ones digits and then multiplied the tens digits like this:</td>
</tr>
<tr>
<td>7*1 = 7</td>
</tr>
<tr>
<td>Then I added two zeros because I covered up two digits and got 700.</td>
</tr>
<tr>
<td>So, Jamie was right – he had spent over $500 on CDs.</td>
</tr>
</tbody>
</table>

Goal:
1) Show you one new way to solve: Talk through how Frank solved the problem – Write 72 * 14 on board, cover up ones digits with hands

Preview next 2 days:
1) Work with a partner each day, studying how other kids have estimated, answering questions about their ways, and estimating on your own.
2) No homework tonight!
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Proximity Introduction Lesson

(Note: All students can receive the same proximity introduction lesson.)

Overall Goal: In about 10 minutes, 1) introduce students to the use of the number line and 2) the idea of proximity or closeness to the exact value as a means to evaluate estimates.

Pass-out Handout with problems and student solutions

Problem 1:

About how much is 12 * 53?

Jay’s way:

12 * 53

My estimate is 530.

I rounded one number.

I rounded 12 down to 10.

Then I multiplied 10 * 53 and got 530.

The exact value is marked below. Mark Jay’s answer on the number line.

<How did Jay solve the problem?>

(1) One way that we might decide whether Jay’s estimate is a good one or not is to see how close it is to the exact value. I used my calculator to find that the exact value of 12 * 53 is 636. You can see that 636 is marked on this number line.

(2) Now let’s put Jay’s estimate on the number line too. Is Jay’s estimate bigger than or smaller than 636? Right, smaller, so we mark 530 on the number line before 636. (Emphasize that it doesn’t matter if you place 530 in some exact location.) (Put 530 on number line, and write “Jay’s estimate” over it <do this on the board>.)
(3) Is Jay’s estimate close to the exact value? <have student explain reasoning>

(4) Let’s say that another student, Amy, found an estimate for this same problem and got an estimate of 1000. Mark where Amy’s estimate on the number line. How do I know where to put a mark for Amy’s estimate? (With your help, students should first note that Amy’s estimate is on the other side of the exact value than Jay’s. Then they should note that it should be a bit farther away than Jay’s. Put 1000 on number line, and write “Amy’s estimate” over it. Note that Amy’s estimate is intentionally large so that it is as obvious as possible that she is farther away from the exact value than Jay is. Don’t worry about explaining to students what strategy Amy might have used to get this estimate – if they ask, just say you don’t know and keep going.)

(5) Is Amy’s estimate close to the exact value? <have student explain reasoning> (Don’t dwell on the comparison between Amy’s and Jay’s estimates. A good answer to this question would be “not that close,” rather than “closer than Jay.”)

(6) Now you are going to continuing working with your partner, as you did yesterday. On some of the problems, you’ll see number lines like the one we used here – these can help you figure out whether estimates are close to the exact value or not, but don’t worry about being exact when you mark the number on the number line. At the end of the class period, we’ll hand out your homework for tonight.
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Wrap-up lesson

(Note: All students can receive the same wrap-up lesson.)

Overall Goal: In about 5 minutes, provide some points of closure about estimation.

Suggested Lesson: Here are the main points that we’ve tried to emphasize in the past three days of problem solving.

I. Remember that
(a) Estimation is a way to get an approximate answer.
(b) There are many ways to arrive at an estimate.
(c) Different ways of estimating will give different estimates

(Note: Just say the three things above, without elaboration or examples; it should take you less than half a minute. Instruction will go more slowly and deliberately through part II below)

II. Let’s think about estimating the answer to your last homework problem: 17 \times 32. (Write problem on board)

(a) Some ways of getting an estimate may be better than others because they are simpler to do. Simpler means that the calculations are easier to do, because they involve easy numbers. If (point to one student) used a different way than (point to another student) to find an estimate for this (point to board) problem, one of your ways might be better because it is simpler or has easier calculations to do in your head and not make mistakes.

(b) Some ways of getting an estimate may be better than others because they get closer to the exact answer. So if two people estimated the answer to this problem (point to board), one might be better because it gets you closer to the exact answer.

\begin{itemize}
  \item Can someone share their estimate to 17 \times 32? \textless do NOT have them share strategy\textgreater
  \item How many people gave a different estimate? \textless raise hands\textgreater
  \item Can someone share a different estimate to 17 \times 32? \textless 600 and 640 are likely estimates\textgreater
\end{itemize}

For 17 \times 32, the exact value is 544. So which estimate is closer?

(c) We can also think about how close an estimate is by figuring out how much it differs from the exact value. For 17 \times 32, if I estimated it as 20 \times 32 \textless write on board\textgreater, I rounded 17 up 3 to 20, so I know I have 3 groups of 32, or 96, more than the exact value \textless write \( 3 \times 32 = 96 \).\textgreater