What to compare? Algebra 2007

Overview of timing:

Pretest Day: Pretest (50 minutes) (Monday, October 15)

Day 1 Intervention: (Tuesday, October 16)
  Distribution intro lesson – 10 min;
  Partner work on Packet 1 (40 min)
  Distribute homework 1

Day 2 Intervention: (Wednesday, October 17)
  Collect Homework 1
  Variable on both side intro lesson – 5 min;
  Partner work on Packet 2 (45 min)
  Distribute homework 2

Day 3 Intervention: (Thursday, October 18)
  Collect Homework 2 (return homework 1?)
  Fractions intro lesson – 5 min
  Partner work on Packet 3 (37 min)
  Wrap-up lesson – 8 min
  Distribute homework 3

Posttest Day: (Friday, October 19)
  Collect homework 3;
  Posttest;
  Give treat

General Information for Intervention Days:

- Have desks in pairs with packets on the desk before class begins so students can sit with their partner as they come in.
- Audiotape your lesson using a small silver digital recorder with your name on it
- If someone’s partner is absent, have them join another pair who has the same color label on their packet (so they are in the same condition). Have them note who their partners are on the cover sheet.
- During partner work, audiotape 3 “official” pairs, one per condition, using the digital recorders labeled for student pairs. This should be the same pairs each day. Use recorder log to note students’ numbers and the folder on the recorder being used. Distribute other small silver CLL digital recorders to pairs as desired (helps keep kids on task), but we will not save these files.
- At end, check students’ work on independent practice problems (we have keys). They must use algebra and show their work. Also check their answer to the guided practice problem. Check off students who complete packets on roster (so students know they are being held accountable).
Contrasting Cases in Algebra 2007

Instruction

Day 1 Lesson: Reminder about distributing and modeling of partner work (10 minutes)

Clip digital recorder (labeled with your name) to your shirt/pants and turn it on to tape your lesson.

We are going to be learning about different ways to solve algebra equations for the next few days, and you are going to be working with a partner a lot. So let’s start by looking at a problem that I’d like you to try to solve. Try to do this problem yourself for just a minute.

(present problem on overhead: $3(x + 1) = 12$; have solution and steps written out ahead of time and just uncover one line at a time.)

There is more than one way to solve this problem. We’ll go through one way to solve it together and see how we are going to label our steps. The first thing I would do is use the distributive property. I would take 3 times x and get 3x, plus, 3 times 1 and get 3, so I’d have 3x + 3. I’m going to label my step “distribute the 3.” What is the next thing I could do? ...

$3(x + 1) = 12$
$3x + 3 = 12$ Distribute 3
$3x = 9$ Subtract 3 on both
$x = 3$ Divide both by 3

You are going to be working with a partner on the problems this week, so I want to show you how you and your partner should work together to solve these problems.

Put the following up on an overhead:

Hanna’s Solution:

$2(y - 3) + 5y = 22$
$2y - 6 + 5y = 22$ Distribute ______
$7y - 6 = 22$ Combine ______
$7y = 28$ Add ______ on Both
$y = 4$ Divide ______ on Both

1. How do you know if Hanna solved this equation correctly?
Here you see how a student named Hanna solved an equation. Let’s imagine that my partner _______ and I are going to try to understand how Hanna solved this equation.

First, ___________ and I should look at Hanna’s solution, try to see what Anna did, and finish labeling her steps.

I: Let’s see, ___________. What did Hanna do first? It says “distribute,” so she must have applied the distributive property here. But what did she distribute?

Partner: She distributed the 2: 2 times y minus 2 times 3 is 6. So we should write “2” in the blank next to distribute. Then what did Anna do? She combined like terms – what did she combine?

I: She combined 2y and 5y to get 7y. So let’s write 2y and 5y in the combine blank. Now what’s next? She was adding something to both sides – what did she add?

Partner: It looks like she added 6 to both sides to get rid of the minus 6. What you do to one side, you have to do to the other to keep it equal. 22 plus 6 is 28. So I’m going to write 6 in the add ___ to both blank. And finally, Anna divided to both sides?

I: Yes, she divided both sides by 7, so let’s write 7 in the divide __ on both blank.

Next, ___________ and I need to answer the questions that are below Hanna’s solution.

I: It says, How do you know if Hanna solved this problem correctly? Hmmm. What do you think, ____________?

Partner: I think that we know this because if we put the answer of 4 back into the original equation, we get...

I: I see where you are heading. Put back in 4 – so 2 times (4 – 3) is 2 times 1 or 2, plus 5 times 4 or 20. So 2 plus 20 is 22, so it works. So I’ll write, “If you put back the 4 into the original equation, the left side and the right side both equal 22, so you know it works.”

So this is how you and your partner will be working in the next few days. You’ll have a packet of equations to work on, and you’ll need to work together to figure out how problems are solved and to answer questions like ___________ and I did. Sometimes you’ll also be asked to solve some problems on your own.

Here is a sheet with the labels for the 4 basic steps that we use when solving equations, just as a reminder. You can use this sheet whenever you’d like in the next few days.

One thing to keep in mind for the next few days: There is more than one way to solve any equation. When you are deciding what way to solve an equation and which of the steps to use, it is important to keep the two sides of the equation equal. So this side (gesture
underneath left side of example on overhead) must stay equal to this side (gesture to other side).

Does anyone have any questions? OK – then we are ready to begin. <can turn off digital recorder>
Day 2 Intro to Variables on Both Sides (5 minutes)

Turn on digital recorder to tape the lesson.

Everyone did a great job yesterday – today we are going to work on a new packet. Some of the problems you’ll see today have variables on both sides, so I wanted to take a few minutes to review these types of equations.

First, let’s look at this problem:

\[ 5x + 6 = 3x + 10 \]

Take a moment and try to solve it yourself first. (Wait a minute.) There are lots of ways that this problem could be solved but here is one way. First we could subtract 3x from both sides, etc.

Go through solution to this problem as a class, including labels for steps:

\[
\begin{align*}
5x + 6 &= 3x + 10 \\
2x + 6 &= 10 & \text{Subtract 3x from Both} \\
2x &= 4 & \text{Subtract 6 from Both} \\
x &= 2 & \text{Divide 2 on Both}
\end{align*}
\]

This problem was a little different than the ones from yesterday because we had x’s on both sides of the equal sign. Here is another example equation. Our first step could be to distribute. What would we get if we distributed? Go through solution to this problem as a class, including labels for steps:

\[
\begin{align*}
15t &= 5(2t - 7) \\
15t &= 10t - 35 & \text{Distribute 5} \\
5t &= -35 & \text{Subtract 10t on Both} \\
t &= -7 & \text{Divide 5 on Both}
\end{align*}
\]

Today you’ll be working on some equations like these with your same partner but with a new packet.

And as you are working, keep in mind that there is more than one way to solve any equation. When figuring out how to solve an equation, think about what steps get you closer to finishing the problem and what makes it less likely that you’ll make a mistake. For example, kids sometimes make errors when they need to distribute, so you could try a way where you don’t need to distribute. Any questions?
Day 3 Intro to equations with fractions (5 minutes)

Turn on digital recorder to tape the lesson.
Nice work on the last couple of days. Today, we’ll start with these warm-up problems.

Warm-up problems for the day (start the class with these on the board):

(1) What is 6 ÷ 1/2?
(2) Solve this equation: 7x = 14

After students have had a chance to work on these for a few minutes, go over each one on the board/overhead:

(1) Did anyone remember how to divide a number by a fraction? That’s right – you invert and multiply. So 6 ÷ 1/2 is the same as 6 * 2/1. And then remember that 6 is the same as 6/1. So we have 6/1 * 2/1. Then you multiply the numerators (point to 6 * 2) and you multiply the denominators (point to 1 * 1) to get 12/1. So our final to 6 ÷ 1/2 is 12. Any questions about how I did this?

(2) I hope this one was easy for you! To get x by itself, we divide both sides by the number in front of x (also called the coefficient of x). So divide by sides by 7. So x is 2. Any questions about how I did this?

Today, we are going to work on some equations that make use of what we just talked about on the warm-ups. Here is an example problem for us to talk about as a class (cover-up solution)

\[ \frac{1}{3}x = 4 \]

\[ x = \frac{4}{1} \div \frac{1}{3} \]  
Divide both by 1/3

\[ x = \frac{4}{1} \times \frac{3}{1} = \frac{12}{1} \text{ or } 12 \]  
(Same as multiplying by 3/1)

To solve this equation, we need to get x by itself. How do we get x by itself? <call on student>

Note: Students may say “multiply by 3 on both sides,” which is also correct. If so, first solve showing multiplying both sides by 3, rather than below <write solution to the side>. Then, note that multiplying both sides by 3 is the same as dividing both sides by 1/3. In packet, we use label “divide” because works for whole numbers and fractions. Dividing is multiplying by the reciprocal, so can think of it either way.)
We divide both sides by 1/3. That’s because we need to divide both sides of the equation by the number that is in front of x (which is called the coefficient).

How do we divide by 1/3? This is the tricky part – to divide by a fraction, we need to multiply by its reciprocal, which looks like this:

On the left side, dividing by \( \frac{1}{3} \) gets rid of the \( \frac{1}{3} \) because 1/3 divided by 1/3 is 1.

On the right side:

\[
\frac{4}{\frac{1}{3}}
\]

is the same as

\[
4 \times \frac{3}{1}
\]

and

\[
\frac{4}{1} \times \frac{3}{1} = \frac{12}{1} \text{ or } 12
\]

and so

\[
x = 12
\]

Let’s do another problem like this:

\[
\frac{1}{4}x + 2 = 6
\]

See if you can solve this problem yourself. (Wait a minute.)

There are lots of ways that this problem could be solved, but here is one way:

*Go through solution to this problem as a class, including labels for steps...*

<table>
<thead>
<tr>
<th>( \frac{1}{4}x + 2 = 6 )</th>
<th>( \frac{1}{4}x = 4 ) Subtract 2 on both</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 16</td>
<td>Divide both by ( \frac{1}{4} )</td>
</tr>
</tbody>
</table>

(which is the same as multiplying both sides by 4) can write “multiply by 4 on both” if this is what kids say.

Today you’ll be working on some equations like these with your same partner but with a new packet.

And as we said yesterday, keep in mind that there is more than one way to solve any equation. When figuring out how to solve an equation, think about what steps get you closer to finishing the problem and what makes it less likely that you’ll make a mistake. Any questions?
Day 3 Wrap-up lesson (8 minutes) (do at end of 3\textsuperscript{rd} intervention day)

*Turn on digital recorder to tape the lesson.*

Everyone has done a great job with the work we’ve been doing for the past several days. Tomorrow, we are going to give you a test to see if the work you’ve been doing has made sense to you. Today, I wanted to go over some of the things that you may have noticed as you’ve been working on solving equations.

(1) *There is more than one way to solve an equation. Any way is OK as long as you always keep the two sides of the equation equal.*

You looked at lots of examples of students’ work as they solved equations, and you noticed that different students solved equations in different ways. A solution is correct as long as you keep the two sides of the equation equal-- by doing the same thing to both sides or by simply combining or distributing terms on one side.

(2) *Some ways to solve an equation are better than other ways.*

When you looked at the examples, you may have noticed that some ways are better than other ways to solve the equations. Here is an example. Let’s say you were given the equation:

\[ 3x - 4 = 5 \]

One way that you might solve this equation is to add 4 to both sides. [Write +4 on both sides of the equation to show this, and then write the result, \(3x = 9\).] Doing this step seems like a good idea, because it gets you the line \(3x = 9\), which means you are almost done solving the equation.

Another way that you might solve this equation is to add 10 to both sides. [Write +10 on both sides of the equation to show this, and then write the result, \(3x + 6 = 15\).] *It is fine to do this step since you did the same thing to both sides.* But, it doesn’t seem like a good idea, because it doesn’t help you get closer to solving the equation.

So there is more than one way to solve this equation, but some ways are better than other ways because they get you closer to being able to finish the problem.

Here’s another example to show this. Let’s say you were given the equation:

\[ \frac{3}{10}x + \frac{7}{10} = \frac{13}{10} \]
One way to solve this equation is to subtract $\frac{7}{10}$ from both sides. [Write $\frac{7}{10}$ on both sides of the equation to show this, and then write the result, $\frac{3}{10}x = \frac{6}{10}$.]

This is a fine first step because you did the same thing to both sides, but there are other ways to solve this equation that you might think are better. For example, what if you first multiply both sides of the equation by 10? [Write $10x$ on both sides, and then write the result, $3x + 7 = 13$.] This is a different way to start solving this equation, and since it got rid of the fractions, you might find this way to be easier or better, or this way might make you less likely to make mistakes.

So sometimes some ways of solving an equation are better because they are easier for you or they make it less likely that you’ll make a mistake.

So to review [have these on an overhead],

1. There is more than one way to solve an equation. Any way is OK as long as you always keep the two sides of the equation equal.

2. Some ways to solve an equation are better than other ways, because they get you closer to being able to finish the problem, because they are faster or easier for you to do, or because they make it less likely that you’ll make a mistake.

Please think about these things when you do your homework tonight.