What to compare?  CC6: Algebra 2008

Pretest Day:  Pretest (50 minutes) – allow simple calculators that do not compute with fractions on all days.

Day 1 Intervention:
- Distribution intro lesson – 10 min;
- Partner work on Packet 1 (remainder of class)
- Distribute homework 1 – let students know the answers are on the back to check themselves, and they must show their work (may work on in class if finish early – for class who usually doesn’t do homework, go ahead and give it out, especially if kids finish early)

Day 2 Intervention:
- Collect Homework 1 (don’t review)
- Fractions intro lesson – 10 min
- Partner work on Packet 2 (stop 10 minutes before class ends)
- Wrap-up lesson – 8 min
- Distribute homework 2 (may work on in class if finish early)

Posttest Day:
- Collect homework 2 (don’t review)
- Posttest (50 minutes;
- Give treat

General Information for Intervention Days:
- We know that kids in different classes have different amounts of time to work on the materials, but we will let this vary by school (as we have before).
- Have desks in pairs with packets on the desk before class begins so students can sit with their partner as they come in.
- Audiotape your lesson using a small silver digital recorder with your name on it
- If someone’s partner is absent, have them join another pair who has the same color label on their packet (so they are in the same condition). Have them note who their partners are on the cover sheet.
- At end, check students’ work on independent practice problems (we have keys).
  - They must use algebra and show their work.
  - Initial packets to show you checked it.
- Collect homework, but do not go over it. Give homework to teacher to grade and record however he/she normally does.
**Contrasting Cases in Algebra 2008**

**Instruction**

**Day 1 Lesson: Reminder about distributing and modeling of partner work (10 minutes)**

*Clip digital recorder (labeled with your name) to your shirt/pants and turn it on to tape your lesson.*

We are going to be learning about different ways to solve algebra equations for the next few days, and you are going to be working with a partner a lot. So let’s start by looking at a warm-up problem that I’d like you to try to solve.

Warm-up problem for the day (*start the class with this on the board*):

1. Use the distributive property to rewrite the expression $3(x + 1)$.

*After students have had a chance to work on this for a few minutes, go over it on the board/overhead:*

Does anyone remember how to distribute?

That's right - you get $3 \cdot x$ plus $3 \cdot 1$. You can write $3 \cdot x$ as $3x$, and then write down the plus sign. And $3 \cdot 1$ equals 3. So you end up with $3x + 3$.

*Note: Jon suggests that you figure out how to leave this problem on the board - don’t erase it - as you do the rest of the intro lesson, but erase before students start the packets.*

Here’s another problem that I’d like you to try to solve. Try to do this problem yourself for just a minute.

*Present problem on overhead: $3(x + 1) = 12$; have solution and steps written out ahead of time and just uncover one line at a time. If you write out multiplication sign, use dot.*

There is more than one way to solve this problem. We’ll go through one way to solve it together and see how we are going to label our steps. The first thing I would do is use the distributive property. I would take 3 times $x$ and get $3x$, plus, 3 times 1 and get 3, so I’d have $3x + 3$. I’m going to label my step “distribute the 3.” What is the next thing I could do? ...

<table>
<thead>
<tr>
<th>$3(x + 1) = 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x + 3 = 12$</td>
</tr>
<tr>
<td>$3x = 9$</td>
</tr>
<tr>
<td>$x = 3$</td>
</tr>
</tbody>
</table>
You are going to be working with a partner on the problems this week, so I want to show you how you and your partner should work together to solve these problems.

*Put the following up on an overhead:*

**Hanna’s Solution:**

\[
2(y - 3) + 5y = 22 \\
2y - 6 + 5y = 22 \\
7y - 6 = 22 \\
7y = 28 \\
y = 4
\]

Distribute ____  
Combined _____  
Added _____ on Both  
Divided by _____ on Both

1. How do you know if Hanna solved this equation correctly?

Here you see how a student named Hanna solved an equation. Let’s imagine that my partner _______ and I are going to try to understand how Hanna solved this equation.

First, ________ and I should look at Hanna’s solution, try to see what Hanna did, and finish labeling her steps.

*I: Let’s see, ___________. What did Hanna do first? It says “distribute,” so she must have applied the distributive property here. But what did she distribute?*

*Partner: She distributed the 2: 2 times y minus 2 times 3 is 6. So we should write “2” in the blank next to distribute. Then what did Hanna do? She combined like terms – what did she combine?*

*I: She combined 2y and 5y to get 7y. So let’s write 2y and 5y in the combine blank. Now what’s next? She adding something to both sides – what did she add?*

*Partner: It looks like she added 6 to both sides to get rid of the minus 6. What you do to one side, you have to do to the other to keep it equal. 22 plus 6 is 28. So I’m going to write 6 in the add ____ to both blank. And finally, Anna divided both sides?*

*I: Yes, she divided both sides by 7, so let’s write 7 in the divide __ on both blank.*

Next, ____________ and I need to answer the questions that are below Hanna’s solution.

*I: It says, How do you know if Hanna solved this problem correctly? Hmm. What do you think, _____________?*
Partner: I think that we know this because if we put the answer of 4 back into the original equation, we get...

I: I see where you are heading. Put back in 4 – so 2 times (4 – 3) is 2 times 1 or 2, plus 5 times 4 or 20. So 2 plus 20 is 22, so it works. So I’ll write, “If you put back the 4 into the original equation, the left side and the right side both equal 22, so you know it works.”

So this is how you and your partner will be working in the next few days. You’ll have a packet of equations to work on, and you’ll need to work together to figure out how problems are solved and to answer questions like ___________ and I did. Sometimes you’ll also be asked to solve some problems on your own.

Here is a sheet with the labels for the 4 basic steps that we use when solving equations, just as a reminder. You can use this sheet whenever you’d like in the next few days.

One thing to keep in mind for the next few days: There is more than one way to solve any equation. When you are deciding what way to solve an equation and which of the steps to use, it is important to keep the two sides of the equation equal. So this side (gesture underneath left side of example on overhead) must stay equal to this side (gesture to other side).

Does anyone have any questions? OK – then we are ready to begin. <can turn off digital recorder>

[Erase problems from the board.]
Day 2 Intro to equations with fractions (10 minutes)

Turn on digital recorder to tape the lesson.
Nice work yesterday. Today, we’ll start with these warm-up problems.

Warm-up problems for the day (start the class with these on the board):

(1) What is 3 • 1/3?
(2) Solve this equation: 7x = 14

After students have had a chance to work on these for a few minutes, go over each one on the board/overhead:

(1) 3 • 1/3. Does anyone remember how to multiply fractions?
That's right - you multiply the numerators and you multiply the denominators. But first we have to write 3 as a fraction. So 3 is the same as 3/1. Now the problem says 3/1 • 1/3.
If we multiply numerators and denominators, we get 3/3. What is 3/3?
Right, it is 1. So 3 • 1/3 is equal to 1. This is a pretty useful thing mathematically - to find two numbers that multiply together to give you 1. Does anyone remember what word we use to talk about these two numbers that multiply to give you 1?
They are called reciprocals, or 1/3 is the reciprocal of 3. Any questions about how I did this?
So what is the reciprocal of 4?
Right, 1/4, because 4 • 1/4 is 1 (quickly show on the board)
And a tricky one - what is the reciprocal of 1/2?
Right, 2. Because 1/2 • 2 is 1 (quickly show on the board.

(2) I hope this one was easy for you! To get x by itself, we divide both sides by the number in front of x (also called the coefficient of x). So divide both sides by 7. So x is 2.
Any questions about how I did this?

[Note: Jon suggests that you figure out how to leave these two problems on the board - don’t erase them - as you do the rest of the intro lesson, but erase before start the packets.]

Today, we are going to work on some equations that make use of what we just talked about on the warm-ups. Here is an example problem for us to talk about as a class (cover-up solution)

\[
\frac{1}{3}x = 4
\]

\[
3 \times \frac{1}{3}x = 4 \times 3 \quad \text{Multiplied by 3, the reciprocal of 1/3, on both} \]
To solve this equation, we need to get x by itself.

Right now it says 1/3x, but we want it to say just x. To do this, the idea is to multiply both sides of the equation by a special number that essentially gets rid of the 1/3 - and this number is the reciprocal of 1/3. What is the reciprocal of 1/3? Right, it is 3 - remember the problem we did a moment ago [point to it]? So if I multiply both sides by 3,

On the left side, we have 3 • 1/3 x. What is 3 times 1/3?

Right, as we did a moment ago [point to it], 3/1 • 1/3 is 3/3, which is 1. And what is 1x? Right, it is just x. So we are just left with x.

On the right side we have 4 times 3, which is 12. so

\[ x = 12 \]

For 1/3x = 4, we could also choose to divide both sides by 1/3. How do you divide by a fraction?
Right, to divide by a fraction, we multiply by the reciprocal, so dividing both sides by 1/3 is the same thing as multiplying both sides by the reciprocal of 1/3, which is 3.

Let’s do another problem like this:

\( \frac{1}{4} x + 2 = 6 \)

See if you can solve this problem yourself. (Wait a minute.)

There are lots of ways that this problem could be solved, but here is one way:

*Go through solution to this problem as a class, including labels for steps...*

\[
\begin{align*}
\frac{1}{4} x + 2 &= 6 \\
\frac{1}{4} x &= 4 & \text{ Subtract 2 on both} \\
x &= 16 & \text{ Multiplied by 4 on both}
\end{align*}
\]

You can see that we chose to multiply both sides by the reciprocal of 1/4, which is 4. If you like dividing by fractions, you could also have divided both sides by 1/4 on the second step [point to it].

Today you’ll be working on some equations like these with your same partner on a new section of the packet.
And as we said yesterday, keep in mind that there is more than one way to solve any equation. When figuring out how to solve an equation, think about what steps get you closer to finishing the problem and what makes it less likely that you’ll make a mistake. For example, kids sometimes make errors when they need to distribute, so you could try a way where you don’t need to distribute. Any questions?

[Erase problems from the board.]
Day 2 Wrap-up lesson (8 minutes) (do at end of 2nd intervention day)

*Turn on digital recorder to tape the lesson.*

Everyone has done a great job with the work we’ve been doing for the past several days. Tomorrow, we are going to give you a test to see if the work you’ve been doing has made sense to you. Today, I wanted to go over some of the things that you may have noticed as you’ve been working on solving equations.

*(1) There is more than one way to solve an equation. Any way is OK as long as you always keep the two sides of the equation equal.*

You looked at lots of examples of students’ work as they solved equations, and you noticed that different students solved equations in different ways. A solution is correct as long as you keep the two sides of the equation equal—by doing the same thing to both sides or by simply combining or distributing terms on one side.

*(2) Some ways to solve an equation are better than other ways.*

When you looked at the examples, you may have noticed that some ways are better than other ways to solve the equations. Here is an example. Let’s say you were given the equation:

\[ 3x - 4 = 5 \]

One way that you might solve this equation is to add 4 to both sides. [Write +4 on both sides of the equation to show this, and then write the result, \(3x = 9\).] Doing this step seems like a good idea, because it gets you the line \(3x = 9\), which means you are almost done solving the equation.

Another way that you might solve this equation is to add 10 to both sides. [Write +10 on both sides of the equation to show this, and then write the result, \(3x + 6 = 15\).] *It is fine to do this step since you did the same thing to both sides.* But, it doesn’t seem like a good idea, because it doesn’t help you get closer to solving the equation.

So there is more than one way to solve this equation, but some ways are better than other ways because they get you closer to being able to finish the problem.

Here’s another example to show this. Let’s say you were given the equation:

\[ \frac{3}{10}x + \frac{7}{10} = \frac{13}{10} \]
One way to solve this equation is to subtract $\frac{7}{10}$ from both sides. [Write $-\frac{7}{10}$ on both sides of the equation to show this, and then write the result, $\frac{3}{10}x = \frac{6}{10}$.

This is a fine first step because you did the same thing to both sides, but there are other ways to solve this equation that you might think are better. For example, what if you first multiply both sides of the equation by 10? [Write $10x$ on both sides, and then write the result, $3x + 7 = 13$.] This is a different way to start solving this equation, and since it got rid of the fractions, you might find this way to be easier or better, or this way might make you less likely to make mistakes.

So sometimes some ways of solving an equation are better because they are easier for you or they make it less likely that you’ll make a mistake.

So to review [have these on an overhead],

(1) There is more than one way to solve an equation. Any way is OK as long as you always keep the two sides of the equation equal.

(2) Some ways to solve an equation are better than other ways, because they get you closer to being able to finish the problem, because they are faster or easier for you to do, or because they make it less likely that you’ll make a mistake.

Please think about these things when you do your homework tonight.