The Benefits of Comparison in Learning to Solve Equations

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AERA poster handout, Chicago, April 2007

Abstract

The sharing and comparing of multiple solution strategies is a central part of current reform efforts in mathematics (Hiebert & Carpenter, 1992; NCTM, 2000). However, experimental studies that conclusively demonstrate the benefits of comparison for student learning are largely absent. The larger project from which this paper is drawn seeks to address this gap by exploring the benefits of comparison for students’ learning of equation solving. The present study reports an analysis of students’ utterances during problem solving, where we looked at the quality of the explanations offered by students who were engaging in comparison.

Introduction

There is a well-established cognitive literature demonstrating the benefits of comparison for student learning. Comparison facilitates deep and meaningful abstraction of information from problems, and prepares students for future learning (e.g., Bransford & Schwartz, 2001). Similarly, comparison is integral to the process of analogical encoding, or the abstraction of common underlying structure from multiple examples (e.g., Gentner, Loewenstein, & Thompson, 2003). In addition, there is evidence from the literature on self-explanations (e.g., Chi, DeLeeuw, Chiu, & LaVancher, 1994) that good learners generate their own explanations to justify steps in a worked example. Furthermore, learners who collaborate with a partner tend to learn more than those who work alone, especially when the partner brings different prior knowledge to the collaboration (e.g., Johnson & Johnson, 1994).

Taken together, prior research indicates the benefits of comparison and also the potential impact of engaging students in self-explanations with peers about multiple, compared worked examples. In this paper (which is taken from a larger project that explored the effectiveness of comparison as an instructional intervention), we look closely at the utterances of student pairs to determine whether differences exist between those pairs who engaged in comparison of worked examples and those who viewed the same worked examples, viewed sequentially.

Hypotheses

We hypothesize that the conversations of student pairs who are learning via comparison will differ from conversations of student pairs who are learning sequentially along two dimensions – the presence of elaborative explanations, and the use of abstract mathematical terminology.

We use the phrase elaborative explanations to refer to verbal statements that indicate that a student pair is trying to understand the reasons behind the problem solving steps used in a worked example.

We use the phrase abstract mathematical terminology to include phrases such as “like terms,” “both sides,” “positive number,” or “variable;” the use of such terms may be linked to greater conceptual knowledge of the domain.

Research questions

Our study explored the following two research questions. First, is comparison of multiple solutions linked to greater use of elaborative explanations? And second, is comparison of multiple solutions linked to greater use of abstract mathematical terminology?

Method

Participants

All 76 7th-grade students (38 females) in a private, urban school participated in the study. Students were from four mathematics classes (3 regular and 1 advanced) and were all taught by the same teacher.

Procedure

We used a pretest-intervention-posttest design. Comparison and sequential conditions were randomly assigned at the classroom level. Students in each condition were randomly assigned to pairs for studying worked examples with their partner. The data collection occurred within students’ intact mathematics classes over five consecutive 45-minute classroom periods.

Materials

Students were given a booklet containing both worked examples and explanation prompts to work on with their partners. Pairs in the comparison condition (n = 20 pairs) studied a pair of worked examples of hypothetical students’ solutions that were put side by side and answered questions that encouraged comparison of the two examples (see Figure 1). Pairs in the sequential condition (n = 18 pairs) studied the same two worked examples on two isomorphic problems that were presented on separate pages and answered questions that encouraged reflection on a single example (see Figure 2).

Analysis

The interactions of the student pairs were transcribed and coded for the presence of elaborative explanations and use of abstract mathematical terms. For coding elaborative explanations, instances of the key phrases “because”, “since”, and “so that” were counted. For the coding of abstract mathematical terminology, instances of the key phrases “like term/s”, “both side/s”, “positive/negative number/s”, “fraction/s”, and “variable/s” were counted.

Results

1. The comparison group made greater use of elaborative explanations

Students from the comparison group had more utterances explaining the strategies they employed or the evaluations they made than students from the sequential group during the intervention period.

Example 1:

One of the questions given in the study asked student

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pairs to explain why combining \(-4(y + 2), 6(y + 2),\) and \(5(y + 2)\) was OK to do. Table 1 shows two student pairs’ utterances, one (A&B) from the sequential group and the other (C&D) from the comparison group. In the sequential student pair’s (A&B) interactions, one student (A) asked his/her partner (B) to explain the use of combine step (combining the three \((y + 2)\) terms), but the student (B) accepted this strategy without giving any explanation. However, in the comparison student pair’s (C&D) interactions, there were three attempts made to explain why it was OK to combine \(4(y + 2), 6(y + 2)\) and \(5(y + 2)\).

| Sequential Pair | A: You know why? You have to explain, you have to explain.  
B: Ok, yes, yes.  
A: Yes, why?  
B: They would combine, anyway. |
|------------------|--------------------------------------------------------------------------------------------------|
| Comparison Pair  | C: Because they both have like terms at each step that they  
D: I guess they both like, they both combining each like terms, which means they both like, like, they are both combining legal things  
C: Yeah, or you can say it’s legal because there is something to combine to it, like you couldn’t do it down here [points to second worked example] because there is nothing you can combine with.  
D: Yeah. |

Table 1: Utterances of two student pairs

Example 2:
Student pairs were asked why it was OK for a hypothetical student James to divide 2 as a first step on both sides of the equation \(2(x - 3) = 8\). Table 2 shows two students’ utterances to their partners, one student from the sequential group and the other from the comparison group. The student from the sequential group did not give any explanation but merely noted the difference between James’ strategy and his own. In contrast, the student from the comparison group explained how James’ way is valid.

| Sequential Student | That’s really weird, he did it different than we were. He skipped the little negative 6.  
Comparison Student | They are both legal because they both are doing it to both sides. |

Table 2: Utterances of two students to their partners

2. The comparison group made greater use of abstract mathematical terminology
Students from the comparison group uttered more abstract mathematical terms than students from the sequential group during the intervention period.

Example 1:
Student pairs discussed a strategy for solving a worked example of the equation \(2(x + 1) + 6(x + 1) = 4(x + 1)\). Table 3 shows two student pairs’ utterances, one (A&B) from the sequential group and the other (C&D) from the comparison group.

| Sequential Pair | A: It says 2?  
B: 2 and 6.  
A: It says a and?  
B: No, 2, 6 and 4. |
|------------------|--------------------------------------------------------------------------------------------------|
| Comparison Pair  | C: OK. They are distributing 2, \(x\), 4, wait, 2, 6 and 4. They’re combining by like terms, they’re subtracting by 4, no, 4x, subtracting 4x. Then subtracting 4. Oh yeah, subtracting 8, then divide by 4. Ok, they subtract, like terms. Then they subtract, 4. And then  
D: Divide by 4. And subtract 1.  
C: OK. They are both subtracting variables. |

Table 3: Utterances of two student pairs

In the sequential student pair’s (A&B) interactions, the students read out the coefficients of the terms \((2, 6\) and 4\) without using any abstract mathematical terms. In contrast, when two worked examples of the same equation were presented, the comparison student pair (C&D) used two abstract mathematical terms in their discussion. They referred to \(2x, 6x\) and \(4x\) (after distribution) in the first worked example and \((x + 1)\) in the second worked example as “like terms”. They also referred to \(4x\) as “variables” instead of merely reading of the symbols.

Example 2:
Student pairs were asked why combining \(-4(y + 2), 6(y + 2),\) and \(5(y + 2)\) is OK to do in a worked example. Table 4 shows two students’ utterances to their partners, one student from the sequential group and the other from the comparison group. The student from the sequential group used the generic and non-mathematical term “stuff” to refer to the terms \((y + 2)\), while the student from the comparison group used “like terms” to refer to the same terms.

| Sequential Student | He combined all the stuff  
Comparison Student | I guess they both like they both combining each like terms. |

Table 4: Utterances of two students to their partners

Conclusion
The results show that comparison is linked to greater use of elaboration explanations and abstract mathematical terminology. One important contribution of this study is that the comparison of multiple solutions can enhance students’ interactions in algebra learning. Another important contribution of this study is that the comparison of multiple solutions can engage students in using formal/abstract mathematical language.

The research reported here was supported by the Institute of Education Sciences, U.S. Department of Education, through Grant R305H050179 to Michigan State University. The opinions expressed are those of the authors and do not represent views of the Institute or the U.S. Department of Education.