

Understanding and improving professional development for college mathematics instructors: An exploratory study

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The past two decades have seen increased concern about the quality of undergraduate mathematics education in the United States. Such concerns stem from low enrolment and retention rates in introductory courses as well as the level of understanding students demonstrate. Improving teaching and learning at the college level has proven to be especially challenging, in part because college faculty traditionally receive little or no formal preparation to teach. Unlike K-12 teachers, college teachers do not participate in preparation programs and there are few if any opportunities for faculty to participate in educational professional development (PD) during their careers.

For most mathematics faculty, being a graduate student teaching assistant (TA) was their first experience with teaching at the college level and the only time when they (may have) participated in PD. Since faculty are unlikely to receive any further guidance regarding their teaching, practices they develop as TAs may well shape their teaching for their careers. Professional development associated with their initial teaching may be the only formal opportunity to help college mathematics instructors develop their teaching practices in effective ways.

This proposal describes the exploratory phase of a project with long-term goals of using findings from K-12 research in the design, implementation, and research of PD for college mathematics instructors. As a first step towards these goals, elements of PD programs with proven results at the K-12 level will be adapted for use with graduate student TAs. The PD will teach TAs about the nature of student understanding of college mathematics concepts and will promote TAs' inquiry into their teaching and their students' learning. During this preliminary phase, data collection methods and materials will be developed and refined. The long-term research goals center on understanding and improving the development of mathematics TAs' teaching practices and examining how development of such practices shape the learning opportunities of students in college courses.

In particular, activities of the proposed exploratory project will include: (1) Design and implementation of a semester-long PD course for mathematics graduate student TAs. Concurrently, TAs will teach undergraduate pre-calculus and calculus courses. (2) Development, piloting, and refinement of PD activities for use as data collection instruments to document TAs' knowledge of student understanding. (3) Development, piloting, and refinement of data collection and analysis methods for documenting TAs' teaching practices and connection of that development to the PD course activities. (4) Development, piloting, and refinement of methods for examining the nature of students' learning opportunities in TAs' classes and changes in those learning opportunities resulting from the PD.

The objective of developing of these methods is to enable investigation of the following research questions: (1) What knowledge and beliefs do TAs possess and how do those factors shape their teaching practices, particularly their planning, instructing, and reflecting? (2) How do TAs engage with PD activities and, as a result of engaging in PD activities, are TAs able to learn about the nature of student understanding of mathematics concepts? Does this learning change TAs' planning, instructing, and reflecting practices? (3) As TAs learn to attend more to issues of student understanding as a result of PD, how is that learning reflected in the learning opportunities created for students?

The preliminary research described in this proposal will inform future work that is likely to have broad impact mathematics education in two areas. First, findings will contribute to the small base of research on teaching and professional development for TAs and research into the nature of student learning in college mathematics courses. Second, the detailed examinations of relationships among teachers' knowledge, beliefs, and practices, (and the relationship between teachers' practices and student learning), will provide deeper insight into and advance theories of teaching and learning useful not only to those who research college mathematics education but also K-12 mathematics teaching and learning.

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Graduate student teaching assistants (TAs) are responsible for a considerable portion of undergraduate mathematics instruction in the U.S. Furthermore, future mathematics faculty come from the current population of graduate students, who are likely to carry teaching practices they develop as TAs into their careers. Consequentially, the professional development (PD) TAs receive has the potential to shape instructional experiences for a substantial number of undergraduates now and in the future. Unfortunately, the research base to help TAs develop as effective teachers is nearly non-existent. Little is known about what TAs are likely to know and believe when they take their first teaching assignments, what their teaching typically looks like, or how best to support development of effective practices.

At the K-12 level, however, there is a base of research about teachers and the development of teaching practices. Although many questions remain, the combination of advances in educational research and the recent mathematics reform efforts have generated powerful findings about how teachers can learn to teach in ways that support student learning in manners advocated by reform. Although reform at the college-level has also occurred (with similar goals as K-12 reform), research in collegiate mathematics education has not advanced to the same extent as it has at the K-12 level. Moreover, the fields of collegiate and K-12 mathematics education research have remained quite isolated from one another and efforts to utilize findings from K-12 research in college teaching and learning are virtually unheard of.

The exploratory work described in this proposal will inform future work on larger projects, which would use findings from research on K-12 teachers and teaching in the design, implementation, and research of PD for college mathematics instructors. This project involves the development, piloting, and revision of activities, data collection and analysis methods, and specific instruments to document TAs' learning from PD activities as well as the learning of their students. Elements of PD programs with proven results at the K-12 level will be adapted for use with graduate student TAs. The PD will teach TAs about the nature of student understanding of college mathematics concepts and will promote TAs' inquiry into their teaching and their students' learning. During this exploratory work, pilot data will also inform the refinement of a research agenda aimed at understanding and improving the development of mathematics TAs' teaching practices and connecting development of such practices to the learning opportunities of students in college courses. Research studies will examine the knowledge and beliefs (related to student understanding of particular mathematics concepts) that TAs bring to their planning, instructing, and reflecting practices, as well as the ways such knowledge and beliefs interact with their learning from PD activities.

Rationale

Over the past two decades, there has been increased concern about the quality of undergraduate education in the United States. Of particular concern has been preparation in mathematics, the physical sciences, and engineering. Concerns stem from low enrolment and retention rates in introductory courses as well as the level of understanding that students demonstrate (Douglas, 1986; NSF, 1986, 1989; Steen, 1987). In addition, contrary to folk wisdom, it appears that attrition is not simply a function of students' inadequate preparation, aptitude, or lack of interest (Seymour & Hewitt, 1994, 1997). In fact, among students who leave mathematics and physical science majors, the most common complaint about their educational experience is poor teaching (stated by 90.2% for students who switched out of the major). Moreover, the number who switch is substantial: approximately 50% of freshman math, physical science, and engineering students change majors before graduation (Seymour & Hewitt, 1994).

Efforts to improve undergraduate mathematics teaching and learning have taken the form of instructional design projects (e.g., new textbooks, computer-based curricula, new forms of assessment), and to lesser extents, research on teaching and student learning at the undergraduate level. These efforts in undergraduate mathematics education have paralleled efforts to improve teaching and learning at K-12

levels (as typified by as typified by the National Council of Teachers of Mathematics *Standards* (NCTM, 2000; 1991; 1989). Teaching in ways that support these goals has proven to be difficult; efforts to provide K-12 teachers with PD to support appropriate change have also faced considerable challenges (Cuban, 1990; Fennema & Scott Nelson, 1997; Sykes, 1990).

Improving teaching practices at the college level has proven to be especially challenging, in large part because college mathematics faculty traditionally receive little or no formal preparation to teach. Unlike K-12 teachers, college teachers do not participate in preparation or certification programs and there are few if any opportunities for faculty to participate in educational PD during their careers (NSF, 1992). This means that opportunities to initiate change are minimal. In fact, for most mathematics faculty, being a TA was their first experience with teaching at the college level and the only time when they (may have) participated in PD. Since faculty are unlikely to receive any further guidance regarding their teaching, the practices they develop as TAs may well shape their teaching for their careers.

The importance of improving college teaching practices becomes especially apparent when one considers TAs' role in the typical undergraduate education. At many colleges and universities, undergraduate students spend as much time in classes taught by graduate student TAs as they do in ones taught by faculty instructors. This comes in many forms: some TAs have sole responsibility for teaching a course, others teach recitation or discussion sections that accompany large lectures given by faculty, and some provide tutoring services to students. Whatever the form, contact with TAs can constitute a significant amount of undergraduates' instructional time. Thus, the potential influences TAs have on undergraduate students' experiences with mathematics are tremendous.

Despite challenges inherent in improving college teachers' practices and the critical role these practices play in undergraduate education, the research base to help TAs develop as effective teachers is nearly non-existent (Gutmann, Speer, & Murphy, 2002; Speer, Gutmann, & Murphy, in press). Literature addressing teaching and learning at the K-12 level includes a rich base of information about how teachers think about mathematics and about the teaching and learning of mathematics. This base is extended to document how teachers practice in and reflect upon their own classrooms, including how knowledge and beliefs shape teaching practices and teachers' learning from PD. In contrast, there is no comparable base of information about TAs, neither about their knowledge and beliefs, nor about how their practices develop, nor about how they understand and think about their own classroom practices. Basic research is needed to guide efforts to help TAs develop their teaching practices in productive ways. In addition, despite the similarity of the issues, findings from work in K-12 are rarely used as a basis for studies at the college level and there is virtually no cross-fertilization between research at the K-12 and college levels (Robert & Speer, 2000).

Project overview

The proposed project begins to address the critical needs outlined above. The work focuses on understanding and improving the development of graduate student TAs' teaching practices and examining how development of such practices shapes their students' learning opportunities. More specifically, the proposed project will examine the following **research questions**:

- What beliefs and knowledge about student understanding and learning do mathematics graduate students possess and bring to their work as college mathematics instructors?
- How do those beliefs and knowledge shape TAs' engagement with and learning from professional development activities designed to increase TA's knowledge of student understanding?
- What do TAs learn from engaging in PD activities focused on knowledge of student understanding? In what ways does that learning change how TAs use such knowledge in their planning, instructing, and reflecting practices?
- How do TAs' teaching practices shape the learning opportunities of their students? How does TA learning, as a result of the PD activities, change students' learning opportunities? In particular, as TAs learn to attend more to issues of student understanding and learning, how is this reflected in the learning opportunities created for students?

These research questions will be pursued in the context of a professional development course for college mathematics instructors. The central **project activities** will include the following:

- Design and implementation of professional development for TAs based on prior research on K-12 professional development.
- Design and implementation of PD activities to elicit TAs' beliefs and knowledge about student understanding and to support TAs to learn about the nature of student understanding and learning of key math concepts.
- Support of TAs as they inquire into their teaching and develop planning, instructing, and reflecting practices that make use of knowledge about student understanding and learning.

From research conducted in the context of the PD, the following **products** are anticipated:

- Studies that integrate investigations of teacher planning, instructing, and reflecting and the roles of knowledge and beliefs in these practices.
- Studies that integrate investigations of teaching practices (planning, instructing, and reflecting) with investigations of student learning.
- Methods for examining teachers' learning while engaged with PD activities tied to the context of their teaching.
- Studies examining differences in knowledge, beliefs, and teaching practice between TAs participating in the PD and those in a matched-comparison sample.

This proposal describes an **18-month plan of development, piloting, and refinement of methods and instruments**. The project will begin with development and piloting of materials and data collection methods. This plan also includes opportunities to collect data related to the research questions. After a period of data analysis and revision, a small-scale version of the larger study will be piloted. Thus, the 18-month period will generate tested and refined methods as well as preliminary data related to the proposed studies.

Specifically, the project will provide PD for mathematics graduate students through a semester-long course. TAs will concurrently teach undergraduate pre-calculus or calculus courses while enrolled in the PD course. The PD course will consist of both in- and out-of-class activities, designed to enhance TAs' knowledge of student understanding and learning and promote inquiry into the nature of their students' learning. The goal of the PD is to provide TAs with experiences that will assist them in developing their knowledge, beliefs, and practices. A secondary goal is for those practices to be generative, meaning that the practices of inquiring into and attending to issues of student understanding will themselves permit TAs to continue to develop their knowledge of student understanding in the context of their teaching, from future interactions with students. Activities will include reading articles on research about student understanding of college mathematics concepts, examining student's written work, and analyzing written and video cases of college teaching and learning. Some of these activities have already been developed and used in other TA PD programs by one of the PIs (Speer, 2000; Heitsch & Speer, 2000).

The influence of TAs' knowledge and beliefs related to student understanding on their planning, instruction, and reflection practices will be examined throughout the semester. Gains in knowledge of student understanding will be evaluated using assessments with the TAs and a matched comparison group of TAs who are not participating in the PD. These will be authentic assessments of the extent to which TAs utilize knowledge and beliefs of student understanding as they plan and teach as part of their ordinary responsibilities as a TA. Student achievement and learning will be compared for classes taught by TAs in the PD course and the matched comparison group.

The next section draws on research literature to provide motivation for the research questions and the project's research design and methods. Details of data collection and analysis methods as well as descriptions of specific instruments are provided in the subsequent section.

Relevant literature

This section describes areas of research that inform the theoretical framing, design, and methods that shape the project's goals and research questions. The current proposal focuses on design and refinement of data collection and analysis methods. Rationale for these methods is tied to the larger project and associated research agenda. Discussion of the literature relative to the larger project is provided below as background and context for the proposed development and piloting work.

Teaching and learning to teach are extremely complex enterprises. From findings in teacher cognition and other sub-fields of educational research, it is clear that many factors influence these processes. Research suggests, however, that knowledge and beliefs are especially significant in teaching and the development of teaching practices. The sections below contain descriptions of what is known about roles of knowledge and beliefs in teaching, learning to teaching, and PD, as well as descriptions of theoretical perspective and methods that most directly inform the proposed research.

Influences of knowledge and beliefs on teaching. Although many factors shape teaching practices, the proposed project focuses on the roles of knowledge and beliefs because of the widely-acknowledged and substantial role these factors play in teaching (Richardson, 1996; Thompson, 1992). Teachers' knowledge has been the object of much examination, over many decades, as evidenced by the abundant references in large reviews of literature (e.g., Borko & Putnam, 1996; Calderhead, 1996; Munby, Russell, & Martin, 2001). Early research presumed that the relevant aspects to attend to were teachers' subject matter knowledge and general pedagogical knowledge. More recent work has broadened this perspective to include other kinds of knowledge, most notably pedagogical content knowledge (PCK) (Grossman, 1990, 1992; Shulman, 1986). PCK is a unique kind of knowledge that intertwines content with aspects of teaching and learning.

Knowledge (taken to mean subject matter, pedagogical, and pedagogical content knowledge) may be what teachers draw on when making decisions, but beliefs shape what teachers see as appropriate, relevant, and possible in their teaching (Richardson, 1996; Thompson, 1992). Beliefs have been described as a "messy construct" with different interpretations and meanings (Nespor, 1987; Pajares, 1992). Current definitions of beliefs found in the mathematics education literature, however, focus primarily on how teachers think about the nature of mathematics, teaching, learning, and students. In this context, beliefs are defined as conceptions, personal ideologies, worldviews, and values that shape practice and orient knowledge (Ernest, 1989; Thompson, 1984, 1985, 1992).

In essence, beliefs shape which aspects of knowledge teachers bring to bear when making teaching decisions (Pajares, 1992), including what teaching routines to enact, what goals to accomplish, and what the important features are of the social context of the classroom. Beliefs shape teachers' practices and hence the learning experiences of students. Teachers' beliefs are therefore vital determinants of the type of understanding students have the opportunity to acquire.

Despite the considerable research into teachers' knowledge and beliefs, many issues remain under-examined. Findings have yet to make substantial contributions to the development and/or refinement of theories that illuminate the underlying processes of teaching. Some studies have identified correlations between teachers' knowledge and beliefs and their teaching practices, but few studies are designed to examine specific aspects of knowledge and beliefs and then trace them to practices such as planning, in-class instructing, and reflecting. *Although connections are presumed to exist, it is not common for studies to directly examine the roles of knowledge and beliefs in teachers' instructional decisions and moment-to-moment teaching practice.* Very few studies have involved the fine-grained and detailed examinations necessary to illuminate these connections, despite calls for such work (Richardson, 1996; Thompson, 1992). In addition, most studies of knowledge, beliefs, and practices have been conducted with elementary teachers. Considerably less is known about the content and role of knowledge and beliefs for other populations of teachers and how generalizable findings and theories might be.

<p>Connection of literature on teaching to proposed project: To address under-examined issues in this area, the proposed project involves development and refinement of methods to investigate teachers' knowledge and beliefs, particularly teachers' <i>use</i> of such knowledge and beliefs in planning, instructing,</p>
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and reflecting. Interview methods will be developed to capture teachers' knowledge and beliefs about student understanding, as they are utilized in practices. In addition, although educational research has made advances in understanding the processes of planning, instruction, and reflection, in most cases, research has examined only one or two of those processes. The methods developed in this project will enable the examination of all three sites of teachers' work.

Influences of knowledge and beliefs on learning to teach. From research on teacher preparation and change it appears that knowledge and beliefs play significant roles in how teaching practices develop initially and evolve over time (Ball, 1988b; Borko & Putnam, 1996; Richardson, 1996; Schifter & Simon, 1992). Influenced by their own experiences as students as well as more formal teacher preparation, teachers form beliefs about what it means to teach, how it is one learns, what the nature of mathematics is, and what students are and should be like. In addition, "experienced teachers' attempts to learn to teach in new ways also are highly influenced by what they already know and believe about teaching, learning, and learners" (Borko & Putnam, 1996, p. 684). Despite this complexity, "A growing body of research suggests that lasting and meaningful changes in teaching practices must be accompanied by changes in the fairly fundamental beliefs that teachers hold about the nature of learners and the learning process" (Borko & Putnam, 1996, p.684).

Although the relationships are complex, change in teaching practices appears tied to change in beliefs and knowledge. No simple cause and effect pattern seems to exist (Grossman, 1992; Guskey, 1986; Kagan, 1992), but there is a growing recognition that change in beliefs and knowledge and change in teaching practices happen in an interconnected, cyclic fashion (Kagan, 1992; Thompson, 1992). With this recognition of the roles of beliefs and knowledge, PD efforts have targeted teachers' beliefs and knowledge, practices, or both.

It has also been shown that beliefs and knowledge play important roles in how teachers engage with and learn from specific PD programs and activities (Darling-Hammond, 1990; Franke, Fennema, & Carpenter, 1997; P. L. Peterson, 1990; Scott Nelson, 1997; Simon, 1997; Sykes, 1990; Wiemers, 1990; Wilson, 1990). In a summary of findings from research into teacher change, Borko and Putnam (1996, p.683) claimed that, "What is increasingly clear is that whenever teachers set out to adopt a new curriculum or instructional technique, they learn about and use the innovation through the lenses of their existing knowledge, beliefs, and practices." Therefore, it is essential that designers of PD acknowledge and address issues related to knowledge and beliefs in the development and implementation of PD activities.

Connection of literature on learning to teach, teacher change, and professional development to proposed project: During the proposed project, methods (described in detail below) will be developed that enable detailed investigations of connections between teachers' knowledge and beliefs, and the development of teaching practices. PD activities that elicit and develop knowledge and beliefs will be refined for use as data collection instruments. These instruments will document TAs' use of knowledge and beliefs as they engage in a variety of planning, instructing, and reflecting activities as part of the PD course. In addition, pre- and post-PD interview protocols will be developed to document changes that occur in TA's use of knowledge and beliefs related to student understanding in their planning and reflecting practices.

Promising perspective on PD: Cognitively Guided Instruction. Professional development programs that report success in enabling K-8 teachers to develop reform-oriented practices share several characteristics. These characteristics include (1) an extended focus on small, but meaningful, aspects of practice, and (2) a recognition that people make sense of new information in light of their existing knowledge, beliefs, and practices. One particularly noteworthy example of a successful program is Cognitively Guided Instruction (CGI), a program that was run out of the University of Wisconsin-Madison in the late 1980's and early 1990's. CGI focused on very specific aspects of practice; integral to CGI's design was the prominent role that teachers' existing knowledge and beliefs played. CGI has achieved a level of success that is not typically reported in the PD literature and has had substantial

research projects associated with it. The Purdue Problem-Centered Mathematics Project (Cobb, Wood, & Yackel, 1990) and SummerMath for Teachers (Schifter, 1993) provide other, similar examples.

In CGI, the goals were for teachers to develop a very small (but substantive) set of new teaching practices that allowed them to focus more extensively on mathematical sense-making in their classrooms. Research on CGI (e.g., Fennema et al., 1996) has demonstrated that teachers can become more knowledgeable of student learning and understanding of mathematical concepts. In the program, teachers were exposed to research on student learning and supported as they investigated student understanding. Gaining such knowledge appears to change teaching practices, resulting in attention being paid to issues of student thinking in instructional decisions.

Research on CGI has also examined student learning, teacher beliefs, and practices. Through observations and interviews, teachers' beliefs and practices were rated as to how "cognitively guided" they were. CGI teachers were significantly more likely than a control group to develop beliefs that instruction should build on students' existing knowledge. Most teachers displayed more cognitively guided instructional practices and beliefs by the end of the program. Student problem solving abilities and understanding of concepts increased over the years teachers participated in the program.

Based on the success of CGI, it appears that when PD focuses on a small set of instructional issues, development and change in knowledge, beliefs, and practices can occur. Building on the success of CGI, other new forms of PD have also emerged to support teachers in developing knowledge, beliefs, and practices necessary to teach in reform-oriented ways (Arcavi & Schoenfeld, 2003; Sherin & Han, 2002). To date the majority of such efforts have involved elementary school teachers. It is not known whether such an approach could be effective with other populations of teachers.

Rationale for adapting CGI for use with college instructors: A CGI-oriented program is well-suited to implementation at the college level for multiple reasons. First, perhaps because of their actual or perceived ability in mathematics, TAs hold often strong, narrow, and frequently implicit beliefs about teaching and learning. Typically TAs view many of the concepts in the courses they teach as "elementary" and therefore straight-forward to teach and learn. Focusing closely on student understanding of key concepts in college mathematics is a promising approach to developing TAs' teaching practices because the PD will involve eliciting and having TAs examine their beliefs in light of inquiry into the nature of actual student understanding. In addition, while TAs may possess rich knowledge of the mathematics concepts they are asked to teach, they are unlikely to have well-developed pedagogical content knowledge (or even be cognizant of the existence of such knowledge). Given findings from K-12 educational research, focusing on the development of pedagogical content knowledge is apt to be particularly productive at the college level. Moreover, given TAs' level of prior education, engaging them explicitly in discussion of mathematical content would likely be met with resistance—whereas discussion and inquiry into *student* understanding of the content is likely to be more engaging and thus more productive.

A challenge to implementation of CGI in other areas has been the availability of relevant research on student understanding. Although research in undergraduate mathematics education is a relatively new field, there now exists sufficient research on student understanding of some key college mathematics topics to make such a program possible. The proposed PD will make use of the research related to four concepts for which literature exists. These **focal concepts** will be **limit**, **derivative**, **integral**, and **function** (as it interacts with student understanding of the other three concepts). Using a CGI-like approach, the proposed PD will center on enriching TAs' knowledge of student understanding of select, focal concepts, through reading of research articles and inquiry into the nature of their own students' understanding. Sample activities are provided in the instrument development section below.

<p>Connection of Cognitively Guided Instruction to proposed project: The proposed project involves piloting a PD program for mathematics TAs based on CGI. The PD will center on enriching TAs' knowledge of student understanding of select, focal concepts, through reading of research articles and inquiry into the nature of their own students' understanding during in- and out-of-class activities. A set of</p>

existing activities (Heitsch & Speer, 2000; Speer, 2000) will be the basis of the PD. Methods based on CGI will be developed, piloted, and refined to collect and analyze data on the extent to which TAs' instructional practices are "cognitively guided." In addition, prior CGI work will be extended to the areas of planning and reflecting by developing methods for capturing and assessing the extent to which those practices are cognitively guided.

Methods for examining the relationship between teaching and student learning. If beliefs and knowledge shape teaching practices, it is natural to assume that links exist between teachers' beliefs and knowledge, and student learning. It also seems reasonable to assume that if teachers learn (resulting in changed knowledge and beliefs), then resulting changes in practices will alter what students learn. However, these links between teacher characteristics and student achievement have been, at best, challenging to document. Recently, attention has turned to alternative conceptualizations of what it means to have knowledge for teaching.

Many studies report teachers' knowledge of mathematics, with discouraging findings:

"This body of research overwhelmingly paints a dismal picture of teachers' conceptual knowledge of the mathematics they are expected to teach. By and large, teachers have a strong command of the procedural knowledge of mathematics, but they lack a conceptual understanding of the ideas that underpin the procedures." (Mewborn, 2003), p. 46).

Evidence suggests, however, that additional coursework in mathematics is not the remedy. It may appear non-trivial to "validate empirically the common maxim that the more mathematical knowledge teachers have, the more mathematical knowledge their students will have" (Ball, Lubienski, & Mewborn, 2001), but in fact, research has generated contradictory and counter-intuitive findings about the relationship between knowledge and teaching. One of the most commonly cited examples of such work is Begle (1979). His findings were surprising: The extent of teachers' mathematics preparation (as measured by courses taken) produced positive main effects on students' achievement in only 10% of cases— and negative main effects in 8% of cases. Other studies have found that people majoring in mathematics had just as much difficulty with concepts and with justifying procedures and solutions as did elementary teachers with less preparation in mathematics (Ball, 1988a, 1990). It does not appear that additional course work in mathematics contributes to teachers' knowledge of mathematics as it relates to teaching.

Recently, new trends in research on teachers' knowledge have emerged. Instead of examining knowledge by counting courses taken or by using other similar measures, some researchers have proposed to focus on teachers' knowledge as enacted in teaching practices (e.g., Ball et al., 2001). This approach to research entails examining the knowledge evidenced by teachers as they engage in authentic practices. Ball, Lubienski, & Mewborn (2001) claim that "what matters ultimately is not only what courses teachers have taken or even what they know, but also whether and how teachers are able to *use* mathematical knowledge in the course of their work" (p. 450). This new development has generated work that shows promise for illuminating the role of teachers' knowledge in teaching. This trend to focus on teachers' use of mathematical knowledge (as well as beliefs and other aspects of knowledge) in the moment-to-moment context of teaching is also echoed in the work of other researchers (Aguirre & Speer, 1999; Schoenfeld, 1998, 1999a, 1999b; Sherin, 1996; Speer, 2001).

To date, a small body of research has examined in-class, instructional practices and the roles of knowledge and beliefs in shaping those practices (Aguirre & Speer, 2000; Schoenfeld, 1998, 1999, 2000; Speer, 2001). Connections among knowledge, beliefs, and teaching are established from fine-grained analyses of videotaped instruction and interviews with teachers about instructional decisions. Such work provides theoretical framework and methods appropriate to the study of teachers' beliefs and knowledge, as used in instructional practices, at a very detailed level. While these developments present new possibilities, many questions remain. In particular, how might investigations of knowledge and beliefs, as used by teachers in the context of teaching, be carried out in varied settings of teachers' work? In particular, how can such investigations be expanded to examine the roles of knowledge and beliefs in the planning, instructing, and reflecting practices? How are characterizations of teachers' knowledge and beliefs related to students' learning opportunities and achievement?

Examining teacher's knowledge and beliefs as they are used in authentic acts of teaching opens up new ways to examine connections between teacher characteristics and student learning. Although such connections have been elusive with earlier characterizations of teachers' knowledge and beliefs, these new approaches that examine knowledge and beliefs in more detail and in the context of teaching, may have the potential to uncover the underlying relationships. It is reasonable to presume that studies examining knowledge and beliefs, as they are actually used by teachers as they interact with students, may help illuminate connections between these factors and the learning opportunities created for students.

Connection of literature on relationship of teaching and student learning to proposed project: The proposed project will utilize a design that permits examination of the entailments that knowledge and beliefs have for teachers' authentic, moment-to-moment instructional decisions. Methods will be developed and piloted for capturing TAs' decision-making criteria (especially their reference to knowledge and beliefs related to student understanding) as they plan, instruct, and reflect on their teaching. These kinds of detailed, contextualized characterizations of teachers' knowledge and beliefs may shed light on the nature of learning opportunities they create for students in ways not previously possible.

Research design and methods

Overview. Mathematics graduate student TAs will participate in a semester-long PD course designed and conducted by the project PIs. Concurrently, all TAs will teach undergraduate courses in pre-calculus or calculus. The PD will consist of in- and out-of-class activities designed to increase TAs' knowledge of student understanding and promote reflection and inquiry. TAs will participate in activities during the seminar portion of the PD course and will conduct investigations in the context of their own teaching. Artifacts of TAs' learning during the PD will be collected and periodic documentation of teaching practices in the concurrently taught course will occur. Before beginning the PD, TAs will complete several pre-test measures assessing their knowledge and beliefs about students' understanding of the mathematical content of the courses they will be teaching. To evaluate the effectiveness of the PD, specifically to distinguish between what one might expect TAs to learn by virtue of teaching and what they might learn from the PD, a matched comparison group is included in the research design. A group of TAs matched in terms of prior courses taught, teaching experience, and current teaching assignment will be selected. This matched comparison group will engage in the same pre- and post-test activities as TAs in the PD, as well as periodic documentation of their planning, instructing, and reflecting practices. As an additional measure of the effectiveness of the PD, data on students' learning, including test and exam scores and problem solving interviews, will be collected in the classrooms of both participant and matched comparison TAs.

The five phases of this pilot project are described below and summarized in Figure 1. The first phase will focus on development and piloting of TA interview protocols, development of student problem-solving interviews questionnaires, and refinement of PD activities for use as data-collection instruments. In particular, this will entail piloting the baseline interview and the Planning, Instructing, and Reflecting (PIR) interview protocols with a small sample of TAs. Also during this phase, student interview protocols and questionnaires will be developed. In Phase Two, data from phase one will be analyzed to inform revision of instruments and to provide initial data relevant to some of the research questions. Student protocols will also be piloted and revised during this time. During Phase Three, the PD course will be offered and TA and student data will be collected. During Phases Four and Five, data will be analyzed and instruments will be further refined. Revised TA and student interview protocols will also be tested again and data obtained will be analyzed to inform research questions and the further revision of protocols in anticipation of offering the PD course again in Fall 2005. Data obtained during Phases One, Three, and Four will be used to generate preliminary findings related to the project research questions and write initial reports and articles.

Setting. The project will be conducted at Michigan State University (MSU). MSU enrolls approximately 35,000 undergraduate students and approximately 7,000 graduate students. There are approximately 400

undergraduate mathematics majors and 120 mathematics graduate students at MSU, producing approximately 40 bachelor's degrees, 15 masters degrees, and about 13 doctorates per year. MSU has been selected as the site of the proposed work primarily for convenience, as both project PIs serve on MSU's faculty, one with a partial appointment in the mathematics department.

Figure 1: Project activity timeline

	Phase One	Phase Two	Phase Three	Phases Four & Five
	Spring 2004	Summer 2004	Fall 2004	Spring/Summer 2005
Foci of work	TA and student interview and questionnaire protocol development and piloting.	Analysis of Spring 2003 data; revision of TA and student interview protocols and questionnaires; refinement of PD activities.	Pilot small-scale version of full study.	Analysis of Fall 2004 data; revision and testing of protocols.
PD course		Develop and refine PD course activities for use as data-collection instruments	Use PD activities as data collection instruments. Collect video data of PD course	Revise PD activities based on analysis of Fall 2004 data.
TAs	Pilot background, baseline, PIR, and post-semester interview protocols.	Analyze interview data and revise protocols.	Collect PD activity data. Collect background, baseline, PIR, and post-semester interview data.	Analyze PD activity and interview data; write reports; revise protocols.
Students	Develop problem-solving interview protocol and student questionnaires.	Pilot and revise problem-solving interviews and student questionnaires.	Collect problem-solving interview and student questionnaire data.	Analyze Fall 2004 data; revise protocols.

Participants. TAs will all be graduate students in MSU's Mathematics Department. In addition to receiving course credit, TAs in the PD course who choose to participate in the project will be compensated \$200. When the PD is offered, TAs of similar teaching experiences and backgrounds will be recruited to serve as a matched comparison group, such that each participating TA will have a matched TA in the comparison group. TAs who are selected to be in the matched comparison group will be compensated \$100 for their participation.

Timeline for methods and instrument development:

Phase One (Spring 2004) — TA and student protocol development and piloting

Activity during this phase will center on developing and piloting the TA protocols and developing student interview and questionnaire protocols.

TA background questionnaire development and piloting

To gather general information about TAs, a background questionnaire will be distributed when the PD course is offered. Questionnaire data will be used to find appropriately matched pairs. The questionnaire will assess number of years teaching experience, prior teaching experiences (which courses), course evaluation scores for courses taught¹, and stage in graduate program (pre-qualifying exam, post-qualifying exam, mid-dissertation). During Phase One, the questionnaire will be piloted and refined to ensure that it will capture sufficient information to enable the creation of matched pairs of TAs (during Phase Three).

¹ Students complete a standard MSU evaluation form at the end of each course, in part to evaluate the instruction they have received. These scores are tabulated and made available to the course instructor and his/her department.

TA baseline interview development and piloting

During this phase, development on the baseline interview protocol will be finished and the protocol will be piloted. Some development is already complete (i.e., selection of focal concepts (described below), earlier pilot of portions of the interviews).

Baseline interviews will be used to establish the extent to which TAs reference knowledge and beliefs about student understanding when talking about tasks and planning lessons. During Phase 3, data from these interviews will be used as a baseline for comparisons of TAs in the PD course and matched comparison groups. Each interview will last approximately two hours, and all interviews will be audiorecorded and transcribed. These assessment interviews will include three strands: a task strand, a lesson planning strand, and video-prompted reflection strand, as described below.

Interview protocol: TAs will be asked to complete a series of tasks, talking out loud about their solution strategies. 2-4 problems appropriate for the course each TA is teaching and involving focal concepts will be selected. TAs will be asked to describe the mathematical ideas in each problem, what they would want students to think about while working on the problem, what students would have an opportunity to learn by working on the problem, and what difficulties or alternative conceptions students might hold related to the mathematical ideas in the problem.

In particular, for each task, TA will be asked a series of questions:

- How difficult do you think this task would be for your students?
- If a student solved the task successfully, what would you hope they understood?
- What do you think typical student solution strategies would be for this task?

What do you think typical difficulties might be for students as they worked on this task?

This portion of the interview will provide data on TAs' knowledge (including content and pedagogical content knowledge) and beliefs about student understanding associated with particular concepts that will be discussed during in the PD. Fine-grained analysis will be performed on interview transcripts. These interview and analysis methods have been used successfully in the past to obtain rich data on teacher's knowledge and beliefs related to specific mathematical tasks (Speer, 2003).

To explore TAs' planning practices, TAs will write a lesson plan designed to teach one of the focal concepts (as perceived by the TA) explored in the task questions. They will be asked questions about their lesson plan, including their reasons for the different parts of the plan and rationale for their choices of problems and activities. Having them explain their thinking and instructional decisions will provide access to aspects of their thinking that may not be reflected in the written plan. Based in part on the approach advocated by Ball et al. (Ball et al., 2001), this portion of the interview provides data on the extent to which TAs use their knowledge and beliefs about student understanding in planning for instruction. Since this portion of the interview is more directly connected to their instructional practices, additional knowledge and beliefs may be cued in this context, providing additional data on their practices.

Finally, the baseline interviews will generate data on TA's reflection practices through discussion of videocase. Videoclips of college mathematics classes will be used to assess what TAs notice and think about while watching instruction, as well as the extent to which TAs attend to issues of student understanding in examples of instructional practice. Learning to notice pedagogically-substantive aspects of instruction is an acquired skill and a core component of developing "professional vision" for teaching mathematics (Sherin, 2001). Supporting teachers to learn to notice new things and to discuss them in new ways has emerged as a promising form of PD and a new indicator of teacher learning (Arcavi & Schoenfeld, 2003; Sherin & Han, 2002).

TA Planning, Instructing, and Reflecting (PIR) interview development and piloting

In addition to the baseline interviews, TAs will also engage in additional assessments designed to trace how knowledge and beliefs about student understanding impact planning, instructing, and reflection in the context of their own teaching. These additional assessments are referred to as the "PIR interview cycle," where PIR stands for **P**lanning, **I**nstructing, and **R**eflecting.

During Phase One, the PIR interview protocol will be developed and piloted with a small set of TAs. When the PD course is offered in Phase Three, the PIR interviews will be conducted several times

during the semester with each TA participant from the PD course and their matched comparison TA. In Phase One, protocols will be developed and piloted. The purpose of these interviews is to examine how TAs make use of knowledge and beliefs about student understanding as they prepare to teach, interact with students in class, and reflect on their teaching. When the PD course is offered, these interviews will be scheduled during the semester at times when the focal concepts from the PD course are a central part of the class or, in the case of pre-college focal concepts, at times when student understanding of the concept is likely to be interacting with student learning of new concepts.

Planning: Similar to the baseline interview described above, this portion consists of interviewing TAs about their plans for a class they will be teaching. TAs will be asked to describe the tasks they are going to use with students and any other aspects of class that they have planned. TAs will be asked about how they think students will respond to the tasks and other things they have planned. For the tasks, they will be asked to describe what they would hope for students to think about while solving the task, and what they think some typical student difficulties might be that they will encounter with the tasks and concepts. This portion of the interview will provide information about the extent to which TAs reference knowledge and beliefs about student understanding as well as the particular nature of those knowledge and beliefs.

Instructing: The class the TA was interviewed about during the Planning portion of the interview will be videotaped. Two videocameras will be used, one focusing on the TA's actions and the other focusing on students' actions. Documenting the TA's instructional practices serves two purposes. First, analyzing the videotapes will provide data on the extent to which the TA makes use of knowledge of student understanding while interacting with students during class. Analysis of the video data will be based on frameworks developed by researchers working with the original CGI project (Fennema et al., 1996; P. Peterson, Fennema, Carpenter, & Loef, 1989). Analysis will generate rating of TAs, indicating how "cognitively guided" their instruction is. The video of class will also be used as an object of discussion during the Reflecting portion of the interview (described below).

Reflecting: Within a few days of each videotaped teaching observation, the PIR cycle concludes with a post-observation "videoclip interview" (Speer, 2001). Prior to the interview, researchers will select excerpts of the class video to use during the interview. These excerpts will include representative interactions between TAs and students as well as examples of times when TAs appear to be relying on knowledge of student understanding in making instructional decision. Examples of times when TAs' lack of knowledge of student understanding surface will also be selected. TAs will be interviewed about the excerpts and asked to describe what they were doing and why during the interaction. The audio recordings of these interviews will be analyzed for evidence of use of knowledge of student understanding and for evidence of times when they appeal to other things when making instructional decisions. As with the data from the pre- and post-semester interviews, such data will be analyzed for what TAs notice and discuss as well as how they discuss it (Sherin & Han, 2002).

The Instructing and Reflecting portions of the interview protocol have been developed and used previously to investigate teachers' beliefs about student understanding (Speer, 2003). The planning portion of the interview is under development, drawing from prior studies that explicitly examined teachers' planning (Arcavi & Schoenfeld, 2003; Sherin & Han, 2002).

Student background questionnaire development

In later phases of the proposed work, student volunteers will be asked to provide a variety of background information, including standardized college entrance exam scores, scores on the MSU mathematics placement exam², high school GPA, grades in previous MSU mathematics courses, and overall MSU GPA. This background information will be used to create a composite measure of prior ability for each student, which will in turn be used to help explore student learning. Questionnaires for collecting this information will be developed during this phase of the project.

² The MSU mathematics department gives all students a mathematics exam during orientation. The results of this exam are used to determine which mathematics courses students should take in the first term.

Student problem-solving interview protocol development

Also, during this phase of the project, problem-solving interview protocols will be developed. Research on cognitively-guided PD similar to the proposed work has found positive relationships between teachers' knowledge and beliefs about their students' thinking and student learning (Carpenter, Fennema, Peterson, & Carey, 1988; Fennema et al., 1996; P. Peterson et al., 1989). As a measure of the effectiveness of the PD course in the present work, student problem-solving achievement will also be carefully investigated. Of particular interest will be whether student develop deeper understanding of key mathematical concepts in classes taught by TAs in the PD as compared to students in the classes taught by matched comparison group TAs, and whether high- and low- achieving students' experience similar benefits. In this phase of the proposed work, problem-solving interview tasks will be selected and protocols will be developed.

Phase Two (Summer 2004) — Analysis of Phase One data, TA and student protocol revision, PD course activity refinement

The focus of activity during this time will be on analysis of the pilot data gathered during Phase One, preparation of PD course activities for use as data-collection instruments, revision of TA and student interview protocols and questionnaires. It will also be possible, if necessary, to re-pilot any of the TA or student protocols during this time.

Data analysis (TA)

One objective of the overall project is to document the extent to which participation in the PD course increases TAs' knowledge of student understanding and TAs' use of that knowledge (and associated beliefs) in their teaching practices. Before this goal can be addressed, it is necessary to determine whether the baseline, PIR, and post-semester interviews yielded sufficiently rich data on TAs' planning, instructing, and reflecting practices. In particular, these methods need to generate data on the extent to which TAs reference knowledge and beliefs about the nature of student understanding as they engage in these teaching-related activities. Therefore, Phase Two analysis will involve looking for evidence of this in the pilot data as well as developing frameworks and criteria for making claims that such data is indeed evidence of TAs' use of their knowledge and beliefs. For the instructing portion, analysis frameworks from the original CGI work will be the basis of the initial work and will be modified as necessary for the particular context of college mathematics instruction. These frameworks will enable the examination of the extent to which a particular TA's instructional practices are "cognitively guided."

Protocol revision (TA)

Based on analyses described above, baseline, PIR, and post-semester protocols will be revised.

Pilot and revise student questionnaire

In Phase Two, the student background questionnaire will be piloted. Although this data collection is anticipated to be relatively straight-forward, this opportunity will be used to ensure that the questionnaire generates data that permits the development of the necessary composite measure of prior ability. Having a reliable and accurate measure of students' prior ability as a control becomes critical in later phases of this project, as well as in future studies.

Pilot and revise student problem-solving interview protocol

Student volunteers will be solicited to participate in the piloting of the two-hour problem-solving interviews. The pilot composite measure of prior ability will be used to select target students, including some with high prior ability in mathematics, and some with low prior ability in mathematics. Interview tasks and prompts will be designed to probe for students' understanding of key mathematical concepts in the course. Each target student will be compensated \$25 for their participation in the problem-solving interview.

Interviews will be transcribed and coded. At issue is whether or not the selected tasks and interview protocols allow for the assessment of students' deep understanding of relevant mathematical concepts, not merely whether or not students complete procedural problems correctly. Tasks which afford a rich

conceptualization of student understanding will be created and/or drawn from existing sources; of particular interest are tasks that enable students to demonstrate that they can: provide reasonable, logical, and conceptual explanations to accompany the use of mathematical procedures; recognize connections or establish relationships between different mathematical topics embedded in problems or solutions; and use multiple strategies for solving problems and recognize when particular strategies may be most appropriate or efficient (Hiebert & Carpenter, 1992; R. R. Skemp, 1978; Star, 2001, 2002; Star & Smith, in press).

Professional development course activities

A set of activities was originally developed by one of the PIs (Speer, 2000) and have been used in six previous offerings TA PD courses at MSU and another institution. The existing activities have not been used explicitly as data collection instruments before. During this phase, course activities will be modified to include more focus on issues of knowledge and beliefs related to student understanding. These modifications will include more explicit requests for TAs to describe their decision-making so that activity artifacts generate rich data on TAs' knowledge, beliefs, and teaching practices. Sample activities are described in the next section.

Phase Three (Fall 2004) — TA PD course, TA and student data collection

During this phase, activities will include: offering the TA PD course; collecting interview data on PD course TAs and their matched comparison TAs; collecting data on the PD activities; collecting data on the PD course meetings; collecting problem-solving interview and questionnaire data from students. The number of TA and student participants will remain small, but all components of the research design will be piloted simultaneously.

In addition to collecting artifacts from all PD activities, all meetings of the PD course will be videotaped to provide additional data on TAs' learning. Student volunteers from all classes taught by TAs in the PD course and those in the matched comparison group will be solicited in the first weeks of the semester to participate in the questionnaire and problem-solving interviews.

Professional development course activities

The PD will be offered as a for-credit course in MSU's Division of Mathematics and Science Education during this phase. The course will meet two hours each week during the 15-week semester. The PD course will have two interconnected themes. The first theme centers on the nature of student understanding and learning of a set of focal mathematics concepts. The second theme is inquiry into the nature of student understanding. TAs will read articles about student learning and understanding of focal concepts and will engage in in- and out-of-class activities that emphasize inquiry into the nature of their own students' understandings of mathematics.

Each activity entails inquiry into some aspect of the TA's teaching. Thus, artifacts TAs produce while engaging in the activities (both in and out of class) will provide additional data on their planning, instructing, and reflecting practices. Data from course activities will be used in conjunction with data from the PIR interview cycles to identify activities that appear to be especially effective in eliciting data on the influences in their planning, instructing, and reflecting practices.

Examples of PD activities. Since one project goal is to utilize existing research on collegiate mathematics education, the PD activities will focus on concepts for which research on student understanding exists. These focal concepts will be **limit, derivative, and integral**, as well as the role that understanding of **function** plays in learning such concepts. To provide a sense of the PD course, three existing activities are described below along with descriptions of data they will generate.

PD activity #1: Reading and using research literature on student understanding. TAs will be given articles on research about student understanding of concepts relevant to the courses they are teaching. Examples of such articles include (Ferrini-Mundy & Graham, 1994; Monk, 1987; Monk & Nemirovsky, 1994; White & Mitchelmore, 1996; Williams, 1991). After reading such articles, TAs will be asked to gather data on their own students' thinking about a math concept, use the frameworks presented in the articles to analyze the data in ways similar to those in the article, write up their analyses, and share their findings with their fellow TAs during one of the PD course meetings. TAs' written analyses as well as

their presentations will provide data on the extent to which they are able to understand and use knowledge of student understanding.

PD activity #2: Documenting student understanding. TAs will read articles that present frameworks of characterizing student understanding, such as instrumental and relational understanding (Skemp, 1978). TAs will collect examples from their teaching when students demonstrated instrumental or relational understanding. TAs will then share examples in class, discuss match of examples to categories, select exemplars (to illustrate categories) from set of examples, and generate criteria for evidence of each kind of understanding. The TAs' initial write-ups of their examples provide information about their capacity to characterize student understanding; the write-ups provide information about what they take as evidence of (the different kinds of) student understanding; the in-class discussions provide information about their abilities to recognize what are and are not examples of the different kinds of understanding; the in-class discussions provide information about what they see as essential criteria for distinguishing between the different kinds of understanding.

PD activity #3: Writing quiz questions, predicting and analyzing results. TAs write quiz questions and predict how successful students will be (percentages of students who answer the questions correctly, almost correctly, or incorrectly). TAs also predict typical (correct and incorrect) methods students will use and typical difficulties they will encounter. TAs then administer quiz questions to students and analyze student solutions. TAs write a reflection, synthesizing their predictions and the actual results. The TAs' written predictions and synthesis papers provide data on their knowledge about student understanding.

TAs will generate various documents as they do these activities throughout the semester. These written responses will be collected and analyzed for two purposes. The first purpose is to gain additional information about the TA's use of knowledge of student understanding in planning and to examine the extent to which they are focusing on issues of student understanding as they reflect on their teaching. The second purpose is tied more directly to the course. Although many of these activities have been used before in similar PD courses, no prior analysis has been done on the nature of TAs' engagement with the activities. For example, there may be some activities and/or mathematics concepts that are especially engaging for TAs and that appear to promote substantial inquiry and reflection.

TA post-semester follow-up interview

This interview will be conducted with each TA near or after the end of the semester. The baseline interview protocol will be repeated with different sets of student tasks. Although the tasks will be different, they will still be organized around the same focal concepts. Data from these interviews will be analyzed to determine the extent to which TAs display and reference knowledge of student understanding when planning for instruction. This information can then be compared to the analysis of the baseline data to determine the extent of gains in knowledge of student understanding. Interview data from TAs in the PD course and their matched comparison TAs will be used to determine the effects of the PD course on TAs' knowledge and beliefs related to student understanding of the focal concepts.

Phases Four & Five (Spring & Summer 2005) — Data analysis, report writing, protocol and activity revision

Data analysis (TA)

Fine-grained analysis of pre- and post-semester interview transcripts will be used to document learning as it is reflected in *changes to what TAs notice* and in *how they discuss* the videocases of teaching. Novice teachers tend to provide only literal descriptions of instructional episodes without analyzing how episodes might illustrate something related to principles of teaching and learning (Nemirovsky, DiMattia, Ribeiro, & Lara-Meloy, 2003; Seago, 2000). With experience and/or PD, teachers are more likely to describe episodes with reference to issues of teaching and learning, connecting specific incidents to more general concepts (Copeland, Birmingham, DeMeulle, D'Emidio-Caston, & Natal, 1994; Hughes, Packard, & Pearson, 2000; Sherin & Han, 2002). Frameworks developed by Sherin and Han (2002) will be used in the analysis of TAs reflections.

Analysis of the PD course activity data will also be used to determine which of the activities (or which kinds of activities) appear to generate the most inquiry and reflection on the part of the TAs. Since

the activities are organized around focal math concepts and the pre- and post-interviews will examine TAs' PIR practices in the contexts of these concepts, it will be possible to determine which of the activities produced the greatest knowledge growth in the TAs. Although the data analysis and protocol revision plans are described in this section, we anticipate that this work will continue into the next phase of the project as well.

Data analysis (student)

Student problem solving interviews will be transcribed and coded, with the goal of identifying the degree to which each student developed conceptual understanding of focal math concepts. As described above, understanding of focal concepts will be based on whether or not each student can provide logical and conceptual explanations to accompany the use of mathematical procedures; recognize connections or establish relationships between different mathematical topics embedded in problems or solutions; and use multiple approaches for solving problems and recognize when particular strategies may be most appropriate or efficient.

Summary

The proposed project will contribute to the mathematics education community's ability to provide effective PD for the next generation of college mathematics instructors. In addition, insights from the proposed research will contribute to the community's understanding of the nature of teaching, learning, and the processes of learning to teach. This project will generate and extensively evaluate a series of PD tasks designed to promote growth of TAs' knowledge related to the nature of student understanding of mathematics and development of beliefs that support effective instructional practices. Interviews with TAs who participate in the PD will yield a better understanding of the relationships among teachers' knowledge, beliefs, and their teaching practices. In addition, student problem-solving interview will illuminate relationships between student opportunities to learn and teachers' instructional practices.

Broader impact, products, and dissemination

It is anticipated that the proposed project will result in the development of activities and analytic methods enabling the documentation of TAs' learning from PD, as well as the learning of their students. The activities and methods that emerge from this exploratory project will be used to design one or more larger scale projects, for which additional funding will be sought.

This exploratory work, in addition to the larger studies it will lead to, will have broader impact on the field of mathematics education by leveraging existing research on K-12 teachers and professional develop to inform work done at the college level, advancing theories and findings through fine-grained studies of the roles of knowledge and beliefs in teaching practices, including extending such studies to teacher's planning, instructing, and reflecting practices. The project's development and refinement of research methods will also contribute to the educational research community's capacity to conduct fine-grained investigation of teaching, the development of teaching practices, and student learning. Products of the proposed research will include publication of studies in peer-reviewed journals such as *Journal for Research in Mathematics Education* and *Research in Collegiate Mathematics Education*. Research findings will be presented at conferences (including educational research conferences as well as national meetings of mathematicians such as the Joint Meetings of the Mathematical Association of America and the American Mathematics Society). After the project has concluded, PIs will also pursue publication of the TA PD course activities and provide workshops on TA PD in mathematics departments.

Contributions to NSF's goals

The proposed project on understanding and improving PD for mathematics teaching assistants falls in Quadrant III: Research on STEM Learning in Educational Settings. The project also integrates with and contributes to NSF's past and continued support of instructional innovations in undergraduate education.

Advisory board

The scope of the proposed project requires expertise in the areas of teacher PD, student learning, and collegiate mathematics education. Although the project PIs have individual experience and expertise in

these areas, they have never tried to integrate all aspects into a one project. The advisory board provides expertise in these areas and in coordinating such work in the context of a larger project and will guide the PIs during both the design and implementation phases of the project.

Sol Friedberg (Professor, Department of Mathematics, Boston College) is founder and director of The Boston College Mathematics Case Studies Project. He has experience designing and disseminating PD materials for mathematics teaching assistants and corresponding teaching guides (Friedberg, et al., 2001a, 2001b).

Miriam Sherin (Assistant Professor, School of Education and Social Policy, Northwestern University) has experience designing and researching PD for mathematics teachers. Her research focuses on teacher cognition and the role of video in teacher learning. She is a principal investigator on "Developing a Learn-While-Teaching Mathematics Curriculum: Stimulating Teacher Reflection on Student Learning. Recent articles appear in *Cognition and Instruction* and the *Journal of Mathematics Teacher Education* as well as in practitioner-based journals such as *Mathematics Teaching in the Middle School* and *Educational Leadership*.

Sandra Wilcox (Associate Professor, Department of Teacher Education, Michigan State University) is a Director of the Mathematics Assessment Resource Service (MARS). Her research focuses on preparation and continuing PD of teachers, and the links among assessment, curriculum, teaching, and student learning.

Richard Hill (Professor, Department of Mathematics, Michigan State University) is Director of Michigan State University's Emerging Scholars Program and conducts research in numerical linear algebra as well as students' transition from high school to college mathematics courses.

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