In recent years, there have been important developments in the investigation of change. In this chapter, we emphasize three. First, there has been a renewed emphasis on the collection of multiwave panel data because experts and researchers alike have recognized that two waves of data are insufficient for measuring change effectively (Rogosa, Brandt, & Zimowski, 1982; Willett, 1988, 1989, 1994). Second, the analysis of multiwave data has been revolutionized by individual growth modeling (Bryk & Raudenbush, 1987; Rogosa et al., 1982). Under this approach, individual changes over time are represented in a statistical model as functions of time, and questions about interindividual differences in change are answered by letting the parameters of this individual growth model differ across people in ways that depend on the person's background, environment, and treatment (Rogosa & Willett, 1985; Willett, 1994, 1997). Third, innovative methodologists have shown how the individual growth modeling approach can be mapped onto the general covariance structure model, providing a flexible new tool for investigating change over time called latent growth modeling (see Willett & Sayer, 1994, 1995, for citations to the technical literature).
In this chapter, we use longitudinal data on self-ratings of alcohol use in grades seven and eight for 1122 teenagers (Farrell, 1994) to demonstrate latent growth modeling. There are three main sections. First, we introduce individual growth modeling and show how the multilevel statistical models required for the investigation of change can be mapped onto the general covariance structure model. Second, we show how a single time-invariant predictor, adolescent gender, can be introduced into the analyses to ascertain whether individual growth in alcohol use differs by gender. Third, we demonstrate how a time-varying predictor of change—the pressure to drink exerted on adolescents by their peers—can be included in the analyses. Here, we ask how change in the original outcome, adolescent’s alcohol use, depends on change in the time-varying predictor. We close with comments on extensions of latent growth modeling to more complex settings.

I. LATENT GROWTH MODELING: THE BASIC APPROACH

To investigate change, you must collect multiwave data on a representative sample of people, measuring the status of each member repeatedly over time. In this chapter, we use multiwave data provided by Professor Albert D. Farrell of Virginia Commonwealth University on the self-reported alcohol use of 1122 adolescents (Farrell, 1994). For each adolescent, alcohol use was reported at the beginning of seventh grade, at the end of seventh grade, and at the end of eighth grade. We define corresponding values of the variable, GRADE, as 7, 7.75, and 8.75 years, respectively. Adolescents used a six-point scale to rate how frequently they had consumed beer, wine, and liquor during the previous 30 days. We averaged each adolescent’s responses to the three items to create a composite self-rating of adolescent alcohol use in Farrell’s original metric. Data were also collected on two predictors of change: (a) time-invariant adolescent gender, and (b) time-varying peer pressure. Gender was coded 1 to indicate when the adolescent was female, 0 otherwise. The peer pressure predictor used an anchored six-point scale to record the number of times each adolescent reported being offered an alcoholic drink by a friend in the previous 30 days (Farrell, 1994). In Table 23.1, we present a subsample of 10 adolescents randomly selected from the full data set. Inspection of subsample information—and the full data set—suggests there is heterogeneity in both entry-level (7th grade) alcohol use and change in alcohol use over the seventh and eighth grades. We present the full-sample mean vector and covariance matrix for all variables in the Appendix.

A. Modeling Individual Change

As a first step, we must choose a level-1 statistical model to represent individual change over time. This model is a within-person regression model
TABLE 23.1 Measurements of Adolescent Self-Reported Alcohol Use, Gender, and Peer Alcohol Use over Three Occasions of Measurement

<table>
<thead>
<tr>
<th>Adolescent ID</th>
<th>Outcome</th>
<th>Predictors of change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Adolescent alcohol use</td>
<td>Peer alcohol use</td>
</tr>
<tr>
<td></td>
<td>Start of seventh grade</td>
<td>End of seventh grade</td>
</tr>
<tr>
<td>0018</td>
<td>1.00</td>
<td>1.33</td>
</tr>
<tr>
<td>021</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>0236</td>
<td>3.33</td>
<td>4.33</td>
</tr>
<tr>
<td>0335</td>
<td>1.00</td>
<td>1.33</td>
</tr>
<tr>
<td>0353</td>
<td>2.00</td>
<td>2.00</td>
</tr>
<tr>
<td>0555</td>
<td>2.67</td>
<td>2.33</td>
</tr>
<tr>
<td>0850</td>
<td>1.33</td>
<td>1.67</td>
</tr>
<tr>
<td>0883</td>
<td>3.00</td>
<td>2.67</td>
</tr>
<tr>
<td>0974</td>
<td>1.00</td>
<td>1.67</td>
</tr>
<tr>
<td>1012</td>
<td>1.00</td>
<td>1.67</td>
</tr>
</tbody>
</table>

*Table includes 10 adolescents selected at random from the full data set.*

relating the outcome—here, self-reported alcohol use—to time, and to a few individual growth parameters. If we decide, for instance, that individual change in self-reported alcohol use from seventh through eighth grades has a linear trajectory, then the level-1 model will contain a predictor representing time and two individual growth parameters: (a) an intercept parameter representing an adolescent’s self-reported alcohol use at the beginning of seventh grade (providing that we set seventh grade as the “origin” of our time axis, as we do below), and (b) a slope parameter representing the adolescent’s rate of change in self-reported alcohol use over the period of observation. On the other hand, if we hypothesize that change in alcohol use over seventh and eighth grades has a curvilinear trajectory, perhaps quadratic, then the level-1 model would contain an additional parameter representing curvature, and so on.

In addition, recall that classical test theory distinguishes observed scores from true scores. This distinction is critical when individual change is investigated, because change in the underlying true scores, rather than change in the observed scores, is the real focus of analytic interest. Measurement error randomly obscures the true growth trajectory from view, but it is the true growth trajectory that is specified as a function of time and that is the focus of research interest.

How do you choose a mathematical function to represent true individual change? If theory guides your model choice, then the individual growth parameters can have powerful substantive interpretations. Often, however,
the theoretical mechanisms governing change are poorly understood. Then, a well-fitting polynomial can be used to approximate the trajectory. When only a restricted portion of the life span has been observed and few waves of data collected, the complexity of the growth model must be limited. Because of this, the use of linear or quadratic functions of time is very popular in individual growth modeling.

One strategy for choosing an individual growth model is to inspect each person’s growth record, by plotting their observed scores against time (see Willett, 1989). This type of individual-level exploration is important when latent growth modeling is being used because good data-analytic practice demands knowledge of the data at the lowest level of aggregation so that anomalous cases can be identified and assumptions checked. In our example, initial data exploration suggested that the natural logarithm of self-reported adolescent alcohol use was linearly related to grade level. Thus, we hypothesized the following level-1 individual growth model for self-reported alcohol use:

\[ Y_{ip} = \pi_{1p} + \pi_{2p} t_i + \epsilon_{ip}, \]

where \( Y_{ip} \) represents the natural logarithm of the self-reported alcohol use of the \( p \)th adolescent on the \( i \)th occasion of measurement (\( i = 1, 2, 3 \)), \( t_i \) represents an adolescent’s grade level minus seven years (yielding values of 0, .75, and 1.75 for \( t_i \) through \( t_3 \), respectively), and \( \epsilon_{ip} \) are the level-1 measurement errors that distinguish the true from the observed self-reported alcohol use on each occasion for each person.

The shape of the hypothesized trajectory depends on the functional form of the growth model and the specific values of the individual growth parameters. The growth model in Equation 1 contains a pair of individual growth parameters representing the intercept, \( \pi_{1p} \), and slope, \( \pi_{2p} \), of each adolescent’s trajectory of log-alcohol consumption. The intercept parameter \( \pi_{1p} \) represents the true (self-reported) log-alcohol use of adolescent \( p \) when \( t_i \) is equal to zero—that is, at the beginning of grade seven, because of the way we have created \( t_i \) from adolescent grade level by subtracting 7. Children whose self-reported alcohol use is higher at the beginning of grade seven will possess higher values of this parameter. The slope parameter \( \pi_{2p} \) represents the change in true (self-reported) log-alcohol use per grade for the \( p \)th adolescent. Children whose use increased most rapidly over grades seven and eight will have the largest values of this parameter. These intercepts and slopes are the central focus of our investigation of change.

**B. Mapping the Individual Growth Model onto the LISREL Measurement Model**

Our purpose in this chapter is to show how the individual growth modeling framework begun in Equation 1 can be mapped onto the statistical frame-
work provided by covariance structure analysis, as operationalized in LISREL. One key facet of this mapping is that the individual growth model can be treated as the LISREL measurement model for endogenous variables, \( Y \). Beginning here, and over the next few pages, we illustrate how the mapping evolves.

In our example, three waves of data were collected on each adolescent’s alcohol use, so each individual’s empirical growth record contains three entries—\( Y_{1p} \), \( Y_{2p} \), and \( Y_{3p} \)—on observed (logarithm of self-reported) alcohol use. Each of these observed values can be represented as a function of time and the individual growth parameters, using Equation 1. By repeating the individual growth model in Equation 1 with suitable values of \( Y \) and \( t \) substituted, we can represent this vector of observed data in matrix form, as follows:

\[
\begin{bmatrix}
Y_{1p} \\
Y_{2p} \\
Y_{3p}
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
1 & t_1 \\
1 & t_2 \\
1 & t_3
\end{bmatrix} \begin{bmatrix}
\pi_{t_1} \\
\pi_{t_2}
\end{bmatrix} + \begin{bmatrix}
\varepsilon_{1p} \\
\varepsilon_{2p} \\
\varepsilon_{3p}
\end{bmatrix}
\]

(2)

The introduction of the vector of zeros on the right-hand side of Equation 2 does not change the equation but will facilitate our mapping of the individual growth model onto the corresponding LISREL \( Y \)-measurement model below. Equation 2 now says, straightforwardly, that every adolescent’s alcohol use on each of the three occasions of measurement is constructed simply by bringing the values of their individual growth parameters—their own intercept and slope—together with an appropriate value of time and then disturbing the observation of true alcohol use by an error that arises as a consequence of the measurement process itself.

Now think about the column of measurement errors in Equation 2 more carefully. This equation states that the level-1 measurement error \( \varepsilon_{1p} \) disturbs the true status of the \( p \)th adolescent on the first occasion of measurement, \( \varepsilon_{2p} \) on the second occasion, and \( \varepsilon_{3p} \) on the third. However, we have so far made no claims about the nature of the distribution from which these errors are drawn. Fortunately, latent growth modeling permits great flexibility in the specification of the level-1 error covariance structure and is not restricted to classical assumptions of independence and homoscedasticity. We do not intend to focus, however, on the flexibility of the level-1 error structure specification at this stage, so we assume in all models that we specify subsequently that the level-1 errors are independently and heteroscedastically normally distributed over time within-person. We can describe this assumption statistically, as follows:

\[
\begin{bmatrix}
\varepsilon_{1p} \\
\varepsilon_{2p} \\
\varepsilon_{3p}
\end{bmatrix} \sim N\left( \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}, \begin{bmatrix}
\sigma_{\varepsilon_1}^2 & 0 & 0 \\
0 & \sigma_{\varepsilon_2}^2 & 0 \\
0 & 0 & \sigma_{\varepsilon_3}^2
\end{bmatrix} \right)
\]

(3)
where the mean vector and covariance matrix on the right-hand side of Equation 3 are assumed to be homogeneous across children. Supplementary data analysis indicated that this assumption was very reasonable.

Once we have specified the individual growth model in this way, individual growth modeling and covariance structure analysis begin to converge. We have suggested that the individual growth model in Equation 2, along with its associated error covariance assumptions in Equation 3, map onto the LISREL measurement model for endogenous variables, \( Y \). In standard LISREL matrix notation, this latter model is simply:

\[
Y = \tau_Y + \Lambda_Y \eta + \varepsilon
\]  

Comparing Equations 2 and 4, we can force the LISREL measurement model for endogenous variables, \( Y \), to contain our individual growth model by specifying the LISREL score vectors in a special way:

\[
Y = \begin{bmatrix}
Y_{1p} \\
Y_{2p} \\
Y_{3p}
\end{bmatrix}, \quad \eta = \begin{bmatrix}
\pi_{1p} \\
\pi_{2p} \\
\pi_{3p}
\end{bmatrix}, \quad \varepsilon = \begin{bmatrix}
e_{1p} \\
e_{2p} \\
e_{3p}
\end{bmatrix}
\]  

and by forcing the LISREL \( \tau_Y \) and \( \Lambda_Y \) parameter matrices to contain known values and constants:

\[
\tau_Y = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix}, \quad \Lambda_Y = \begin{bmatrix}
1 & t_1 \\
1 & t_2 \\
1 & t_3
\end{bmatrix}
\]  

Notice that substitution of the vectors and matrices from Equations 5 and 6 into the general \( Y \)-measurement model in Equation 4 leads back to the original individual growth modeling specification in Equation 2. Finally, to represent the level-1 error covariance structure in Equation 3, we let the LISREL error vector \( \varepsilon \) be distributed with zero mean vector and covariance matrix \( \Theta_e \), where:

\[
\Theta_e = \begin{bmatrix}
\sigma_{\varepsilon_1}^2 & 0 & 0 \\
0 & \sigma_{\varepsilon_2}^2 & 0 \\
0 & 0 & \sigma_{\varepsilon_3}^2
\end{bmatrix}
\]  

The advantage of recognizing that the individual growth model and the LISREL \( Y \)-measurement model are one and the same is that the covariance structure specification is quite general and can be expanded to cover more complex analytic settings. If more waves of data were available, for instance, then the empirical growth record on the left-hand side of Equation 2 would contain additional elements and we would simply tack rows onto the \( \Lambda_Y \) matrix, one per occasion of measurement, to accommodate the new waves. Or, if we wished to adopt a more complex trajectory for
individual growth, then we would simply add the appropriate curvilinear terms to the individual growth model in Equation 1. This change would add one or more individual growth parameters representing the curvilinearity to the \( \eta \)-vector in Equation 4 and additional columns to the \( \Lambda_y \) matrix in Equation 6.

Notice that, unlike more typical covariance structure analyses, the LISREL \( \Lambda_y \) parameter matrix in Equation 6 contains only known times and constants rather than a collection of unknown factor-loading parameters that you then try to estimate. Although unusual, this “constants and values” specification forces the critical individual-level growth parameters that are the focus of the investigation of individual change, \( \pi_{1p} \) and \( \pi_{2p} \), into the LISREL endogenous construct vector \( \eta \), which we subsequently refer to as the “latent growth vector.” This allows us to conduct between-person analyses of the individual growth parameters—the level-2 analyses of multi-level modeling—by modeling person-to-person variation in the latent growth vector in the so-far unused LISREL structural model. We describe this next.

C. Modeling Interindividual Differences in Change

Even though we have hypothesized linear change in log-alcohol use over time, everyone need not have an identical trajectory. The trajectories can still differ from person to person because of variation in the values of the growth parameters; some adolescents may have different intercepts; some may have different slopes. These interindividual differences in the growth parameters can be modeled by assuming that each adolescent draws his or her latent growth vector from a bivariate normal distribution, as follows:

\[
\begin{bmatrix}
\pi_{1p} \\
\pi_{2p}
\end{bmatrix} \sim N\left(\begin{bmatrix}
\mu_{\pi_1} \\
\mu_{\pi_2}
\end{bmatrix},
\begin{bmatrix}
\sigma_{\pi_1}^2 & \sigma_{\pi_1\pi_2} \\
\sigma_{\pi_2\pi_1} & \sigma_{\pi_2}^2
\end{bmatrix}\right)
\]

(8)

Conceptually, this is like bobbing for apples. Equation 8 says that, before they begin to change, each adolescent “dips” into a distribution of individual intercepts and slopes, and pulls out a pair of values for themselves. Although these values are unknown to us, they determine the shape of the individual’s change trajectory over time. Because each adolescent draws a different value for their intercept and slope from the same underlying distribution, each person can possess a unique growth trajectory. The parameters of the distribution in Equation 8 then become the focus of our subsequent analyses of change.

The hypothesized distribution in Equation 8 is a simple between-person or level-2 model for interindividual differences in true change. In the current example, in which there are two important individual growth parameters
for each adolescent (i.e., intercept and slope), the shape of the distribution specified in the between-person model is determined by five important level-2 parameters: two means, two variances, and a covariance. These parameters describe interesting features of average change and variability in change in the population, and are worthy of estimation in an investigation of change:

- The two mean parameters, $\mu_{\tau_1}$ and $\mu_{\tau_2}$, describe the average population intercept and slope, and answer the research question, What is the population trajectory of true change in self-reported adolescent (log) alcohol use through grades seven and eight?
- The two variance parameters, $\sigma^2_{\tau_1}$ and $\sigma^2_{\tau_2}$, summarize population interindividual differences in initial (grade seven) true self-reported (log) alcohol use and true self-reported rate of change in (log) alcohol use, and answer the research question, Is there between-person heterogeneity in the growth trajectory of adolescent alcohol use in the population?
- The covariance parameter, $\sigma_{\tau_1,\tau_2}$, represents the population association between initial status and rate of change and answers the research question, Is there a non-zero correlation between initial status and rate of change in adolescent self-reported (log) alcohol use, over grades seven and eight?

D. Mapping the Model for Interindividual Differences in Change onto the LISREL Structural Model

How can we estimate these important between-person parameters in an analysis of change? Fortunately, there is another mapping of the change framework onto the covariance structure framework that helps out. The level-2 distribution of the individual growth parameters in Equation 8 can be modeled via the reduced LISREL structural model, which is

$$\eta = \alpha + B\eta + \zeta$$

The important distribution of the individual intercept and slope, $\pi_{1p}$ and $\pi_{2p}$, in Equation 8 can be modeled in the LISREL structural model because this model is responsible for the modeling of the distribution of the $\eta$-vector, which we have forced to contain the individual growth parameters. The specification is straightforward. We free up the LISREL $\alpha$-vector to contain the population average values of the individual intercepts and slopes and constrain the LISREL $B$-matrix to zero, as follows:

$$\alpha = \begin{bmatrix} \mu_{\tau_1} \\ \mu_{\tau_2} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
This forces the LISREL structural model to become:

\[
\begin{bmatrix}
\pi_{1p} \\
\pi_{2p}
\end{bmatrix} =
\begin{bmatrix}
\mu_{\pi_1} \\
\mu_{\pi_2}
\end{bmatrix} +
\begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\pi_{1p} \\
\pi_{2p}
\end{bmatrix} +
\begin{bmatrix}
\xi_{1p} \\
\xi_{2p}
\end{bmatrix}
\]  
(11)

and, critically, the elements of the LISREL latent residual vector, \( \zeta \), in Equation 11 then contain deviations of the individual intercepts and slopes, \( \pi_{1p} \) and \( \pi_{2p} \), from their respective population means. This means that the covariance matrix \( \Psi \) of the latent residual vector, \( \zeta \), will contain the level-2 variance and covariance parameters that represent the interindividual differences in change from Equation 8:

\[
\Psi = \text{Cov}(\zeta) =
\begin{bmatrix}
\sigma_{\zeta_1}^2 & \sigma_{\zeta_1\zeta_2} \\
\sigma_{\zeta_2\zeta_1} & \sigma_{\zeta_2}^2
\end{bmatrix}
\]  
(12)

By estimating the vector of latent construct means in \( \alpha \) and the latent residual covariance matrix, \( \Psi \), in an empirical application of LISREL, we can address the research questions about interindividual differences in change cited in the previous paragraph.

Although this application appears complex and mathematical, the central idea is conceptually clear. We can force the multilevel models necessary for the measurement of change into the general framework provided by covariance structure analysis. In Equations 1, 2, 3 and 8, the individual growth modeling framework provides level-1 (within-person) and level-2 (between-person) models to represent our hypotheses about the changes underlying the three waves of panel data in our data-example. Equations 4 through 7 and 9 through 12 show that these models can be rewritten using the format and notation of the general LISREL model. Carefully choosing the specification of the LISREL parameter matrices transforms the LISREL Y-measurement model into the individual growth model (which includes all of our assumptions on the distribution of the measurement errors), and the LISREL structural model becomes the level-2 model for interindividual differences in true change. In analyses that follow, we refer to the covariance structure model for interindividual differences in change in Equations 4–7 and 9–12 as Latent Growth Model #1.

Figure 23.1 presents a path diagram of Latent Growth Model #1. The figure illustrates that, by constraining the LISREL \( \lambda \)-coefficients (the factor loadings linking \( Y \) to \( \eta \)) to take on either the value of 1 and or the values of the times of measurement, \( t_1 \), \( t_2 \), and \( t_3 \), we force the latent construct, \( \eta \), to become the individual growth parameters at the center of our change analysis. Then, level-2 relations among these individual intercepts and slopes can be modeled as associations among the latent residuals, \( \zeta \), as indicated by the double-headed arrow on the left of the path model.

Completion of model-mapping allows us to use covariance structure
analysis to obtain maximum likelihood estimates of the parameters of interest in our analysis of change in adolescent alcohol use. All of the critical parameters describing interindividual differences in change now reside in either the \( \alpha \)-vector, the \( \Theta \)-matrix or the \( \Psi \)-matrix, respectively. We can program LISREL to estimate these matrices and thereby answer our questions about change in self-reported alcohol use for these adolescents. In Table 23.2, we provide a LISREL VIII program suitable for fitting Latent Growth Model #1, in which the \( \tau \), \( \Lambda \), \( \Theta \), \( \alpha \), \( \beta \), and \( \Psi \) parameter matrices are patterned as described above.

The program for Latent Growth Model #1 has a structure typical of most LISREL VIII programs. Data are input, variables labeled and selected for analysis, and the latent growth model in Equations 5–7, 10–12 is specified. First, examine the level-1 model for individual change. In the model statement (MO) of lines 7 and 8, the LISREL score vectors are dimensioned according to Equation 5. Notice that the \( Y \)-vector has three elements (\( NY = 3 \)), one for each of the repeated measurements on each adolescent, and the \( \eta \)-vector has two elements (\( NE = 2 \)), one for each individual growth parameter (intercept and slope, labeled \( \pi_{1p} \) and \( \pi_{2p} \) in lines 9 and 10) in the linear growth model. The \( \tau \)-vector has its elements set to zero (\( TY = ZE \)), as required by Equation 6. The factor-loading matrix \( \Lambda \) from the \( Y \)-measurement model is first declared “full” with elements “fixed” to zero (\( LY = FU,FI \)) and is then loaded, in lines 11 through 14, with the values required by Equation 6—that is, the value “1” enters the entire first column.
TABLE 23.2 LISREL VIII Program for Fitting Latent Growth Model #1

Raw data input
1. DA NI = 7
2. RA FI = C:\DATA\ALCOHOL.DAT

Variable labeling and selection
3. LA
4. FEM LN_ALC1 LN_ALC2 LN_ALC3 LN_PEER1 LN_PEER2 LN_PEER3
5. SE
6. 2 3 4 /

Model specification
7. MO NY = 3 NE = 2 TY = ZE LY = FU,FI TE = SY,FI C
8. AL = FR GA = ZE BE = ZE PS = SY,FR
9. LE
10. PI PI2
11. VA 1 LY(1, 1) LY(2, 1) LY(3, 1)
12. VA 0.00 LY(1, 2)
13. VA 0.75 LY(2, 2)
14. VA 1.75 LY(3, 2)
15. FR TE(1, 1) TE(2, 2) TE(3, 3)

Creation of output
16. OU TV RS SS ND = 5

*Line numbers and comments are for reference, and should be removed when the program is executed.

(line 11), and the times of the occasions of measurement, measured as deviations from grade seven (i.e., 0.00, 0.75, 1.75) enter the second column (lines 12–14). This ensures that the elements of the η-vector represent the required individual growth parameters, \( \eta_{i1} \) and \( \eta_{i2} \). The covariance matrix of the level-1 measurement errors, \( \Theta_{ii} \), is then declared symmetric and fixed to zero (TE = SY,FI) in line 7, but its diagonal entries are freed for estimation later in line 15, as required by Equation 7.

The shape of the level-2 model for interindividual differences in change is also specified in the program. In line 8, the shapes of the critical level-2 parameter matrices that make up the LISREL structural model are specified. The \( \alpha \)-vector is freed (AL = FR) to contain the mean values of the individual intercepts and slopes across all adolescents in the population, as is required by Equation 10. The \( \mathbf{B} \)-matrix is forced to zero (BE = ZE) also as a requirement of Equation 10. The covariance matrix of the latent residuals, \( \Psi \), is declared symmetric and freed (PS = SY,FR) so that it can contain the population variances and covariances of the individual growth parameters, as required by Equation 12. Notice that we also set the elements of the \( \Gamma \)-matrix to zero, as it is not used in the current analysis. Finally, line 16 of the program defines the output required.

Execution of the LISREL program in Table 23.2 provides model good-
ness-of-fit statistics, maximum likelihood estimates of the population means and variances of the true intercept and true rate of change in (self-reported) alcohol use, the covariance of the true intercept and true rate of change, and approximate p-values for testing the null hypothesis that the associated parameter is zero in the population (Bollen, 1989, p. 286); see Table 23.3. We have added a brief conceptual description of each parameter. Latent Growth Model #1 fits the data well ($\chi^2 = .049$, df = 1, $p = .826$).

The first two rows of Table 23.3 contain estimates of the population means of true intercept (.226, $p < .001$) and true slope (.036, $p < .001$) from the fitted $\alpha$-vector. They describe the average trajectory of true change in self-reported log-alcohol use over the seventh and eighth grades in this population. Adolescents report consuming statistically significant (i.e., non-zero) amounts of alcohol in seventh grade (average log-alcohol use is reported as .226). The positive sign on the statistically significant mean slope estimate suggests that adolescents report consuming more alcohol as they age. Because we have used a natural logarithm to transform the outcome prior to growth modeling, we can interpret the magnitude of the slope as a percentage increase in alcohol use over time. Therefore, given a slope estimate of .036, we conclude that in this population there is,

**TABLE 23.3** Parameter Estimates for Latent Growth Model #1, in Which the Individual Initial Status (Intercept) and Rate of Change (slope) Parameters of the Alcohol Use Trajectory Differ across Adolescents

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Label</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population true mean trajectory in alcohol use</td>
<td>$\mu_{1}$</td>
<td>Initial status: average true log-alcohol use in grade seven</td>
<td>.226***</td>
</tr>
<tr>
<td></td>
<td>$\mu_{2}$</td>
<td>Rate of change: average true change in log-alcohol use per grade</td>
<td>.036***</td>
</tr>
<tr>
<td>Population true residual variances and covariance in alcohol use</td>
<td>$\sigma_{1}^{2}$</td>
<td>True initial status: variance in true log-alcohol use in grade seven</td>
<td>.087***</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{2}^{2}$</td>
<td>Rate of true change: variance in true change in log-alcohol use per grade</td>
<td>.020***</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{12}$</td>
<td>True initial status and rate of true change: covariance of true log-alcohol use in grade seven and true change in log-alcohol use per grade</td>
<td>-.013***</td>
</tr>
<tr>
<td>Level-1 measurement error variance</td>
<td>$\sigma_{1}^{2}$</td>
<td>At time-1 (beginning of grade seven)</td>
<td>.049***</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{2}^{2}$</td>
<td>At time-2 (end of grade seven)</td>
<td>.076***</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{3}^{2}$</td>
<td>At time-3 (end of grade eight)</td>
<td>.077***</td>
</tr>
</tbody>
</table>

** = $p < .01$, *** = $p < .001$. $\chi^2(1, 1122) = 0.05$ ($p = .83$).
on average, about a 4% increase in self-reported alcohol use per year of adolescence (the exact yearly increase is actually $e^{.0360} - 1$, or 3.66%).

The third and fourth rows of Table 23.3 give the estimated variances of true intercept and true slope from the fitted $\Psi$ matrix, and summarize population interindividual heterogeneity in true change. Because both variances are statistically significant and therefore nonzero, there are important interindividual differences in seventh grade self-reported log-alcohol use and in rate of change in log-alcohol use in the population. In other words, there is recognizable variation in the trajectories of alcohol use across adolescents in the population, and we can legitimately seek predictors of this variation in subsequent analyses.

The fifth entry in the table is also from the fitted $\Psi$ matrix and provides an estimate of the covariance of true intercept and true rate of change in log-alcohol use across adolescents. The product–moment correlation coefficient between intercept and slope is equal to their covariance ($-0.013$) divided by the square root of the product of their variances (i.e., $\sqrt{(0.087 \times 0.020)} = \sqrt{(0.0174)} = .0417$). Thus, the true rate of change has a correlation of $-.31$ with true intercept—a moderate, but statistically significant, correlation that suggests that adolescents with lower self-reported alcohol use in the seventh grade have more rapid rates of increase as they age.

Finally, the sixth through eighth rows of Table 23.3 provide estimates of the error variances on the three occasions of measurement, from the diagonal of the fitted $\Theta$, matrix. Although these variances are of similar magnitude across time, there is some evidence of heteroscedasticity, particularly between times 1 and 2.

II. INTRODUCING A TIME-INVARIANT PREDICTOR OF CHANGE INTO THE ANALYSIS

Once we know that interindividual differences in change exist, we can ask whether this heterogeneity can be predicted by characteristics of the people under study. In our example, for instance, we can investigate whether statistically significant heterogeneity in true change in self-reported (log) alcohol use depends on the time-invariant predictor of change, adolescent gender (FEM). In the following analyses, we investigate whether variation in individual change (i.e., in the intercept and slope of the adolescent alcohol-use trajectory) is related to gender, and address the following research questions: Does seventh grade self-reported (log) alcohol use differ for boys and girls? and Does the rate at which self-reported (log) alcohol use changes over time depend upon gender? Our analysis requires that the gender predictor be incorporated into the previously specified structural model where level-2 interindividual differences in change were modeled.
Within the general LISREL framework, insertion of predictors into the structural model is achieved via the mechanism of the measurement model for exogenous predictors, \( \mathbf{X} \). You load up the selected predictors of change into the \( \xi \)-vector via the X-measurement model and then take advantage of the so-far unused \( \Gamma \) matrix in the structural model, which contains parameters describing the regression of \( \eta \) on \( \xi \) to represent relations between the individual growth parameters and predictors of change. For interpretive purposes, we also recommend centering all time-invariant predictors on their averages as part of this process. This can be achieved simultaneously in the X-measurement model, where any predictor can be partitioned into its mean and a deviation from its mean using a mathematical tautology, as follows:

\[
FEM_p = \mu_{FEM} + 1(FEM_p - \mu_{FEM}) + 0, \quad (13)
\]

where \( \mu_{FEM} \) is the population average of the time-invariant predictor, FEM. Equation 13 says that FEM—and, in fact, any predictor—can be decomposed into two parts: an average, \( \mu_{FEM} \), and a deviation from the average, \( (FEM_p - \mu_{FEM}) \). This representation can then again be mapped directly onto the LISREL measurement model for exogenous variables \( \mathbf{X} \), which is:

\[
\mathbf{X} = \mathbf{r}_x + \mathbf{A}_x \xi + \delta \quad (14)
\]

Comparing Equations 13 and 14, we see that the tautology in Equation 13 containing the predictor of change can be regarded as a LISREL measurement model for exogenous variables \( \mathbf{X} \) with LISREL score and error vectors defined as follows:

\[
\mathbf{X} = [FEM_p], \quad \xi = [FEM_p - \mu_{FEM}], \quad \delta = [0], \quad (15)
\]

with constituent \( \mathbf{r}_x \) and \( \mathbf{A}_x \) parameter matrices:

\[
\mathbf{r}_x = [\mu_{FEM}], \quad \mathbf{A}_x = [1] \quad (16)
\]

and with the covariance matrix \( \Phi \) containing only the variance of the predictor of change:

\[
\Phi = \text{Cov}(\xi) = [\sigma^2_{FEM}] \quad (17)
\]

Notice that, having centered the predictor of change, FEM, on its population average in Equation 13, we estimate that average by freeing the \( \mathbf{r}_x \) vector. Although we do not demonstrate it here, the X-measurement model for exogenous variables in Equations 13–17 can be modified to accommodate multiple time-invariant predictors of change and multiple indicators of each predictor construct, if available. In each case, the parameter matrix \( \mathbf{A}_x \) is expanded to include the requisite loadings (under the usual identification requirements, see Bollen, 1989).

Once the X-measurement model has been specified in this way, the
LISREL structural model then lets us model the relation between the individual growth parameters and the predictor of change, by permitting the regression of $\eta$ on $\xi$. The level-1 individual growth model described in Equations 1–7 is unchanged. However, we express the association between the individual growth parameters and the predictor of change by modifying the existing LISREL structural model in Equations 9–12 so that the newly defined vector of exogenous predictors (now containing adolescent gender, centered on its own mean) is introduced into the right-hand side of the model. We do this by utilizing the latent regression-weight matrix $\Gamma$ present in the LISREL structural model for modeling the association between the $\eta$ and $\xi$ vectors. We free those elements of the $\Gamma$ matrix that represent the simultaneous linear regression of true intercept and slope on the predictor of change, as follows:

$$
\begin{bmatrix}
\pi_{1p} \\
\pi_{2p}
\end{bmatrix} =
\begin{bmatrix}
\mu_{s_1} \\
\mu_{s_2}
\end{bmatrix} +
\begin{bmatrix}
\gamma_{s_1, FEM} \\
\gamma_{s_2, FEM}
\end{bmatrix}
\begin{bmatrix}
FEM_p - \mu_{FEM} \\
0
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\begin{bmatrix}
\pi_{1p} \\
\pi_{2p}
\end{bmatrix}
+ 
\begin{bmatrix}
\xi_{1p} \\
\xi_{2p}
\end{bmatrix}.
$$

(18)

which is the general LISREL structural model:

$$
\eta = \alpha + \Gamma\xi + B\eta + \zeta.
$$

(19)

with constituent parameter matrices:

$$
\alpha =
\begin{bmatrix}
\mu_{s_1} \\
\mu_{s_2}
\end{bmatrix},
\Gamma =
\begin{bmatrix}
\gamma_{s_1, FEM} \\
\gamma_{s_2, FEM}
\end{bmatrix},
B =
\begin{bmatrix}
0 \\
0
\end{bmatrix}.
$$

(20)

It is the LISREL $\Gamma$-matrix that then contains the parameters of most interest in an investigation of change. These regression parameters indicate whether the individual intercepts and slopes that distinguish among the change trajectories of different adolescents are related to adolescent gender. In other words, they tell us whether the trajectory of alcohol use in adolescence differs for boys and girls. We refer to the covariance structure model described by Equations 13 through 20 as Latent Growth Model #2.

In Figure 23.2, we extend the Figure 21.1 path model to include FEM as an exogenous time-invariant predictor of change. On the right-hand side, the factor loadings linking $\mathbf{Y}$ and $\eta$ remain fixed at their earlier values, forcing the $\eta$-vector to contain the individual growth parameters, $\pi_{1p}$ and $\pi_{2p}$. To the left, the newly selected time-invariant exogenous predictor, FEM, predicts the level-1 intercept and slope constructs, as our research questions require. Finally, the latent residuals, $\zeta$, now capture the variation remaining in the intercepts and slopes, after what can be predicted by FEM has been accounted for.

It is worth thinking about the properties of the latent residuals, $\zeta$, in Latent Growth Model #2. In a regular regression analysis, the residual
FIGURE 23.2 Path model for Latent Growth Model #2, in which self-reported log-alcohol use is a linear function of adolescent grade, and initial status (intercept) and rate of change (slope) depend linearly on gender.

variance equals the variance of the outcome when there are no predictors in the model. Once predictors are added, residual variance declines in proportion to predictor effectiveness. The two elements of the latent residual vector $\xi$ in Equations 18 and 19 now contain the values of intercept and slope deviated from their conditional means (i.e., from their values predicted by their linear relation with FEM$_p$). These are the “adjusted” values of true intercept and slope, after partiailling out the linear effect of the predictor of change (i.e., those parts of true intercept and slope that are not linearly related to FEM$_p$). The latent residual vector $\xi$ is therefore distributed with zero mean vector and a covariance matrix $\Psi$ that contains the partial variances and covariance of true intercept and slope, controlling for the linear effects of the predictor of change. If we predict true intercept and slope by FEM$_p$, successfully, then these partial variances will be smaller than their unconditional cousins in Equation 12.

Maximum-likelihood estimates of the new parameters in regression-weight matrix $\Gamma$, along with estimates of other unknown parameters in $\Phi$, $\alpha$ and $\Psi$—which characterize our hypotheses about any systematic interindividual differences in change in the population—can again be obtained using LISREL. We provide a LISREL VIII program for fitting Latent Growth Model #2 in Table 23.4, specifying the $\Lambda_p$, $\Theta_v$, $\tau_v$, $\Lambda_v$, $\Phi$, $\alpha$, $B$, $\Gamma$ and $\Psi$ matrices as defined above. Lines 1–5 of the program are identical to the program listed in Table 23.2. Specification of the Y-measurement model is also identical.
TABLE 23.4  LISREL VIII Program for Fitting Latent Growth Model #2a

Variable selection
6. 2 3 4 1 \n
Model specification
7. MO NY = 3 NE = 2 TY = ZE LY = FU,FI TE = SY,FI C
8. NX = 1 NK = 1 LX = FU,FI TX = FR TD = ZE PH = SY,FR C
9. AL = FR GA = FU,FR BE = ZE PS = SY,FR 
10. LE
11. P11 P12
12. LK
13. Female
14. VA 1 LY(1, 1) LY(2, 1) LY(3, 1)
15. VA 0.00 LY(1, 2)
16. VA 0.75 LY(2, 2)
17. VA 1.75 LY(3, 2)
18. VA 1 LX(1, 1)
19. FR TE(1, 1) TE(2, 2) TE(3, 3)

a Lines 1–5 and 20 are identical to Table 23.2, line numbers and comments are for reference only, and must be removed when the program is executed.

Differences between the programs begin in line 6, where the predictor of change, FEM, is selected for analysis (and labeled as a construct in lines 12–13). In the model (MO) statement (line 8), the new measurement model for X is defined. First, the X and ξ scoring vectors are dimensioned to contain a single element (NX = 1, NK = 1), as required by Equation 15. The factor-loading matrix for X, ΛX, is fixed (LX = FU,FI) and its value set to 1 in line 18 (VA 1 LX(1,1)) as noted in Equation 16. The X mean vector, τX, is freed up to contain the population average value of FEM, as required by Equation 16 (TX = FR). The covariance matrix of X is freed (PH = SY,FR) so that the variance of the predictor of change, FEM, can be estimated, as required by Equation 17. Finally, all measurement errors in X are eliminated (as required by the assumption of infallibility embedded in Equation 13) by setting the covariance matrix of the errors in X, Θx, to zero (TD = ZE).

The structural model is mainly defined in line 9. The population means of the individual growth parameters are freed for estimation in the α-vector, a vector that contains the means of the endogenous constructs, η (AL = FR). The newly important Γ matrix is also freed (GA = FU,FR), as required by Equation 20, to describe the regression of the individual intercepts and slopes on the predictor of change. The B and Ψ matrices have their earlier specifications. The B matrix is set to zero as it is not being used (BE = ZE), and the Ψ matrix is freed for estimation of the latent residual partial variances and covariance (PS = SY,FR).
Table 23.5 contains selected goodness-of-fit statistics, parameter estimates, and p-values for Latent Growth Model #2. Estimates of the level-1 error structure have been omitted to save space. The model fits well ($\chi^2 = 1.54$, df = 2, $p = .46$). The first two rows present estimates of the population means of true intercept and slope from the $\alpha$-vector, which are similar to those obtained under Latent Growth Model #1. Because we have centered the predictor of change on its own mean in Equations 13, 15, and 16, estimates of the average intercept and slope have their earlier interpretation and describe the log-alcohol use trajectory for the average adolescent (not for a male adolescent, as would be the case if no centering had been employed). This trajectory has statistically significant intercept and slope, indicating that adolescents report nonzero alcohol consumption in grade seven and that this consumption rises about 3.6% per grade subsequently.

Rows 3 through 5 of Table 23.5 contain the estimated partial variances
and covariance of true intercept and true slope, controlling for the linear effects of adolescent gender, from the fitted $\Psi$ matrix. Comparing the estimated conditional variances with their unadjusted cousins in Table 23.3, the inclusion of the predictor of change has reduced unexplained variance in true intercept and slope by a very small amount, .8% and 1.5%, respectively, suggesting that gender is a relatively unimportant predictor of interindividual differences in true change in the population. This is further confirmed by statistical tests on the latent regression coefficients, $\gamma_1, FEM$ and $\gamma_2, FEM$, listed in rows 6 and 7, where we reject the null hypothesis associated with predicting the intercept at the .10 level but cannot reject the null hypothesis associated with predicting the slope. This suggests that girls report consuming less alcohol initially than boys in grade seven, but that their rate of change in self-reported consumption is indistinguishable from boys.

III. INCLUDING A TIME-VARYING PREDICTOR OF CHANGE IN THE ANALYSES

Once you recognize that the multilevel models required for investigating change can be mapped onto the covariance structure model, a conceptual portal is opened though which other possibilities come into view. In some research projects, for instance, data are collected on time-varying predictors of change, such as “peer pressure” in our example.

When time-varying predictors of change are available, several different kinds of research questions can be addressed. For instance, the presence of a time-varying predictor allows us to model individual change in both the outcome and the predictor and then to investigate whether changes over time in the two variables are related. In the current example, we can ask, Does adolescents’ self-reported use of alcohol increase more rapidly over the seventh and eighth grades if the pressure exerted on them to drink by their peers is also increasing more rapidly? This asks whether the rate of change in alcohol use is predicted by rate of change in peer pressure. Questions like these can also be addressed with covariance structure methods.

To determine whether change in the outcome depends on change in a time-varying predictor, we model individual growth in both the outcome and the predictor simultaneously and investigate whether individual growth parameters representing change in the outcome can be predicted by individual growth parameters representing change in the predictor. In the current example, this requires that we specify individual growth models for both alcohol use and peer pressure. We proceed in the usual way. First, we specify the individual change trajectory of the outcome (self-reported alcohol use) in the $Y$-measurement model, forcing the individual growth parameters
that describe the true trajectory into the \( \eta \) vector, as before. In previous analyses, we then used the \( X \)-measurement model to pass the newly centered predictor of change into the \( \xi \) vector and ultimately into the LISREL structural model. When incorporating a time-varying predictor, however, we use the \( X \)-measurement model to represent individual change in the time-varying covariate (peer pressure) and force its individual growth parameters into the \( \xi \) vector instead. Then, as before, the relation between \( \eta \) and \( \xi \) is modeled via the matrix of latent regression parameters, \( \Gamma \), which then contains coefficients describing relations between the two kinds of change. (In the example that follows, we have removed the adolescent gender predictor from contention, for simplicity. However, both time-invariant and time-varying predictors of change can be included in the same analysis by combining the two approaches).

In the current example, individual change over time in self-reported (log) alcohol use is modeled in the \( Y \)-measurement model as before (Equations 1–7), but, now we also represent the natural logarithm of peer pressure, \( X_{ip} \), on the \( p \)th child on the \( i \)th occasion of measurement by its own individual growth model:

\[
X_{ip} = \omega_{1p} + \omega_{2p} t_i + \delta_{ip},
\]

where the child’s grade has been recentered on grade seven and \( \delta \) represents measurement error. Again, exploratory graphical analyses suggested that Equation 21 was an appropriate model for individual change in peer pressure over adolescence. In Equation 21, slope \( \omega_{2p} \) represents change in true self-reported (log) peer pressure per grade for the \( p \)th adolescent; teenagers who reported that peer pressure increased the most rapidly over seventh and eighth grades will have the largest values of this parameter. Intercept \( \omega_{1p} \) represents the true self-reported (log) peer pressure on adolescent \( p \) at the beginning of the seventh grade, given our recentering of the time metric; children who report experiencing greater peer pressure at the beginning of seventh grade will possess higher values of this parameter.

As before, we can develop a matrix representation of each adolescent’s empirical growth record in both the outcome and the predictor. The former is unchanged from Equation 2, the latter is:

\[
\begin{bmatrix}
X_{1p} \\
X_{2p} \\
X_{3p}
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} +
\begin{bmatrix}
1 & t_i \\
1 & t_2 \\
1 & t_3
\end{bmatrix}
\begin{bmatrix}
\omega_{1p} \\
\omega_{2p}
\end{bmatrix} +
\begin{bmatrix}
\delta_{1p} \\
\delta_{2p} \\
\delta_{3p}
\end{bmatrix}
\]

(22)

where, as before, we assume that the measurement errors, \( \delta \), are heteroscedastic but independent over time:

\[
\begin{bmatrix}
\delta_{1p} \\
\delta_{2p} \\
\delta_{3p}
\end{bmatrix} \sim N \left( \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix},
\begin{bmatrix}
\sigma^2_{\delta_1} & 0 & 0 \\
0 & \sigma^2_{\delta_2} & 0 \\
0 & 0 & \sigma^2_{\delta_3}
\end{bmatrix} \right)
\]

(23)
As previously, we note that the empirical growth record in self-reported peer pressure in Equation 22 can be mapped onto the LISREL X-measurement model:

$$X = \tau_x + \Lambda_x \xi + \delta,$$  \hspace{1cm} (24)

with LISREL score vectors that contain the empirical growth record, the individual growth parameters, and the errors of measurement, respectively:

$$X = \begin{bmatrix} X_{1p} \\ X_{2p} \\ X_{3p} \end{bmatrix}, \quad \xi = \begin{bmatrix} \omega_{1p} \\ \omega_{2p} \end{bmatrix}, \quad \delta = \begin{bmatrix} \delta_{1p} \\ \delta_{2p} \\ \delta_{3p} \end{bmatrix}$$  \hspace{1cm} (25)

the elements of the $\tau_x$ and $\Lambda_x$ parameter matrices contain known values and constants:

$$\tau_x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \Lambda_x = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ 1 & t_3 \end{bmatrix}$$  \hspace{1cm} (26)

and the error vector $\delta$ is distributed with zero mean vector and covariance matrix $\Theta_\delta$:

$$\Theta_\delta = \text{Cov}(\delta) = \begin{bmatrix} \sigma_{\delta_1}^2 & 0 & 0 \\ 0 & \sigma_{\delta_2}^2 & 0 \\ 0 & 0 & \sigma_{\delta_3}^2 \end{bmatrix}$$  \hspace{1cm} (27)

When we investigate the association between growth in an outcome and growth in a predictor, both the $Y$- and $X$-measurement models are in use, and we have the additional option to specify not only the covariance matrices of the level-1 errors, $\Theta_\delta$ and $\Theta_\epsilon$, but also the matrix of their covariances, $\Theta_{\delta\epsilon}$. In this particular example, this is useful for both substantive and psychometric reasons. In Farrell's survey, both the adolescent's alcohol use and the peer pressure were self-reported on similar instruments and similar scales on each of the three occasions of measurement. We therefore assume that the level-1 measurement errors in self-reported alcohol use and peer pressure covary across adolescents within occasion, as follows:

$$\Theta_{\delta\epsilon} = \text{Cov}(\delta \epsilon) = \begin{bmatrix} \sigma_{\delta_{t_1}} & 0 & 0 \\ 0 & \sigma_{\delta_{t_2}} & 0 \\ 0 & 0 & \sigma_{\delta_{t_3}} \end{bmatrix}.$$  \hspace{1cm} (28)

Having specified the level-1 structure, we can predict the growth parameters describing change in alcohol use by the growth parameters describing change in time-varying peer pressure, by utilizing the LISREL structural
model. These predictions are modeled in the usual way in the following level-2 model:

$$
\begin{bmatrix}
\pi_{1p} \\
\pi_{2p}
\end{bmatrix} = \begin{bmatrix}
\mu_{\pi_1} \\
\mu_{\pi_2}
\end{bmatrix} + \begin{bmatrix}
\gamma_{\pi_{1u_1}} & \gamma_{\pi_{1u_2}} \\
\gamma_{\pi_{2u_1}} & \gamma_{\pi_{2u_2}}
\end{bmatrix} \begin{bmatrix}
\omega_{1p} \\
\omega_{2p}
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix} \begin{bmatrix}
\pi_{1p} \\
\pi_{2p}
\end{bmatrix} + \begin{bmatrix}
\xi_{1p} \\
\xi_{2p}
\end{bmatrix},
$$

(29)

which we again recognize as the LISREL structural model:

$$
\eta = \alpha + \Gamma \xi + B \eta + \xi
$$

(30)

with parameter matrices:

$$
\alpha = \begin{bmatrix}
\mu_{\pi_1} \\
\mu_{\pi_2}
\end{bmatrix}, \Gamma = \begin{bmatrix}
\gamma_{\pi_{1u_1}} & \gamma_{\pi_{1u_2}} \\
\gamma_{\pi_{2u_1}} & \gamma_{\pi_{2u_2}}
\end{bmatrix}, B = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}.
$$

(31)

It is again the $\Gamma$ matrix that contains the important level-2 regression parameters describing relations between an adolescent's growth in alcohol use and growth in peer pressure (i.e., describing relations among the individual growth parameters describing change in predictor and outcome).

In the previous section, we had to account for the population mean and variance of the predictor of change, FEM. In this expanded time-varying-covariates analysis, we must ensure that there is a place in the model for the population distribution of the intercept and slope of the predictor peer-pressure trajectory. Clearly, trajectories of peer-pressure can differ across adolescents, and so their individual growth parameters will possess population averages, variances, and covariances. These parameters will be forced to be zero, underlining the overall fit of the model, if we do not provide them with a place to reside within the general LISREL model. The natural place to model the population average trajectory of the time-varying covariate—that is, $\mu_{u_1}$ and $\mu_{u_2}$—is in the $\kappa$-vector, which records the means of the exogenous construct, $\xi$:

$$
\kappa = \begin{bmatrix}
\mu_{u_1} \\
\mu_{u_2}
\end{bmatrix}
$$

(32)

and the associated variability in $\xi$ is modeled in the covariance matrix of exogenous constructs, $\Phi$, which contains the variances and covariance of the individual intercepts and slopes that now describe change in peer pressure over time:

$$
\Phi = \text{Cov}(\xi) = \begin{bmatrix}
\sigma_{\eta_{u_1}}^2 & \sigma_{\eta_{u_1u_2}} \\
\sigma_{\eta_{u_1u_2}} & \sigma_{\eta_{u_2}}^2
\end{bmatrix}
$$

(33)

In Figure 23.3, we extend the path model of Figure 23.1 to include peer pressure as a time-varying predictor of change. The right-hand side
FIGURE 23.3 Path model for Latent Growth Model #3, in which both self-reported log-alcohol use and log-peer pressure are linear functions of adolescent grade, and the initial status (intercept) and rate of change (slope) of the alcohol-use trajectories depend linearly upon the initial status (intercept) and rate of change (slope) of the peer pressure trajectory.

of the figure is unchanged from earlier analyses, and continues to represent adolescents' change in log-alcohol use over time. However, a similar path structure has been introduced on the left-hand side of the figure to represent change in peer pressure for those same adolescents. Single-headed arrows linking individual growth parameters for change in peer pressure to individual growth parameters for change in alcohol-use provide the hypothesized links between the two kinds of changes. The latent residuals continue to sop up any between-person variation in the alcohol-use trajectory that remains after what can be predicted by change in peer pressure has been removed. We refer to the model in Figure 23.3 as Latent Growth Model #3.

In Table 23.6, we provide a LISREL VIII program for fitting Latent Growth Model #3 to data. The model is specified as in Equations 1–7 and 22–32. Lines 1–5 are identical to those in the program in Table 23.2. In line 6, three waves of data in both log-alcohol use and log-peer pressure are selected for analysis. The Y-measurement model is specified as before in program lines 7, 10, 11, 14, 16–18, and 19, but modifications have been made to both the X-measurement model and the structural model.

The X-measurement model is specified in program lines 8, 15, 16–18, and 20. In the model (MO) statement (line 8), the X and \( \xi \) vectors are dimensioned (NX = 3, NK = 2) to reflect the three waves of longitudinal
data that are available on adolescent peer pressure and the pair of individual growth parameters that are required to represent its trajectory of change, respectively. The new intercepts and slopes of the peer-pressure trajectory are labeled in lines 12 and 13. The X-measurement model’s factor-loading matrix, $A_x$, is first specified as “full” and “fixed” in line 8 ($LX = FU, FI$), but then its elements are given the values required by Equation 26 in lines 15 through 18. The X-mean vector, $\mu$, is set to zero in line 8 ($TX = ZE$), also as required by Equation 26. The mean-vector, $\mu$, and covariance matrix, $\Theta$, which contain the level-2 parameters that describe the between-person distribution of the peer-pressure trajectories, are freed for estimation in line 9 ($KA = FR$) and in line 8 ($PH = SY, FR$), as required by Equations 32 and 33. The covariance matrix of the measurement errors in X, $\Theta_x$, is first declared symmetric and fixed in line 8 ($TD = SY, FI$), and then its diagonal elements are freed for estimation in line 20, as required in Equation 27. Similarly, the matrix containing the covariances of the measurement errors of Y and the measurement errors of X, $\Theta_{xy}$, is declared full and fixed in line 8 ($TH = FU, FI$), and then its diagonal elements are freed for estimation in line 21, as required by Equation 28.

Finally, the new structural model is described (line 9). It is similar to the earlier structural model specified for Latent Growth Model #2. The population means of the individual growth parameters describing change in the outcome, adolescent log-alcohol-use, are freed for estimation in the
\( \alpha \)-vector as before (AL = FR), as required by Equation 31. The \( \Gamma \) matrix is freed (GA = FU, FR), as required by Equation 31, to describe the regression of change in alcohol-use on change in peer pressure. Specification of the \( \beta \) and \( \Psi \) matrices are identical to the earlier specifications. The \( \beta \) matrix is set to zero because it is not being used in this analysis (BE = ZE), and the \( \Psi \) matrix is freed for estimation of the latent residual partial variances and covariance (PS = SY, FR).

Selected parameter estimates and goodness-of-fit statistics are provided for the Latent Growth Model #3 in Table 23.7, along with approximate \( p \)-values. We have omitted estimates of the level-1 error structure in order to save space. The model fits reasonably well—although the magnitude of the \( \chi^2 \) statistic (11.54) is larger than we would like, it is not exorbitant given the 4 degrees of freedom.

A variety of conclusions can be reached by inspecting the table. First, the intercept (.188, \( p < .001 \)) of the average trajectory of log-peer pressure over seventh and eighth grades indicates that adolescents experienced statistically significant pressure from peers to consume alcohol in seventh grade, and the slope (.096, \( p < .001 \)) indicates a statistically significant increase in peer pressure during the period of observation. In fact, because we took the natural logarithm of peer pressure prior to growth modeling, we can interpret the slope estimate (.096) as indicating that adolescents experienced about a 10% increase in peer pressure per year. There is also statistically significant heterogeneity in the peer-pressure change trajectories among adolescents. Rows #8 and #9 of Table 23.7 reveal that the variances of initial log peer pressure (.070, \( p < .001 \)) and rate of change in log peer pressure (.029, \( p < .01 \)) are both statistically significant (i.e., nonzero), but we cannot reject the null hypothesis that they are unrelated (.001, n.s.).

In a similar fashion, rows 1 through 5 of Table 23.7 suggest conclusions about changes in alcohol use. We must interpret these entries cautiously because the estimated values are conditional on the value of the predictor, growth in peer pressure. In other words, the estimated values of \( \mu_{t1} \) and \( \mu_{t2} \) in Latent Growth Model 3 are those for individuals who have “null” trajectories—trajectories with zero intercept and zero slope on the time-varying covariate. However, the results suggest that change in peer pressure is moderately successful in predicting change in alcohol use. Notice, from rows 11–14 of Table 23.7, that two out of the four regression coefficients linking change in peer pressure and change in alcohol use are statistically significant. These coefficients indicate that the initial level of peer pressure is positively related to the initial level of alcohol use (.799, \( p < .001 \)), and that rate of change in peer pressure is positively related to rate of change in alcohol use (.577, \( p < .001 \)), respectively. We can then conclude that adolescents drink more on entry into 7th grade if they have peers who drink more, and that adolescents report more rapid growth in alcohol usage if they experience more growth in peer pressure to drink. This incontrovert-
TABLE 23.7 Parameter Estimates for Latent Growth Model #3, a Model Testing the Impact of Changes in Time-Varying Peer Pressure on the Adolescent's Alcohol Use Trajectory

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population true mean alcohol use trajectory for those with a “null” peer</td>
<td></td>
</tr>
<tr>
<td>alcohol use trajectory</td>
<td></td>
</tr>
<tr>
<td>$\mu_{e_1}$ Initial status: average true log-alcohol use in grade seven</td>
<td>.067***</td>
</tr>
<tr>
<td>$\mu_{e_2}$ Rate of true change: average true change in log-alcohol</td>
<td>.008</td>
</tr>
<tr>
<td>use per grade</td>
<td></td>
</tr>
<tr>
<td>Population true residual variances and covariance in alcohol use</td>
<td></td>
</tr>
<tr>
<td>controlling for change in peer alcohol use</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{e_1/\text{PEER}}^2$ True initial status: partial variance of true</td>
<td>.042***</td>
</tr>
<tr>
<td>log-alcohol use in grade seven</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{e_2/\text{PEER}}^2$ Rate of true change: partial variance of true</td>
<td>.009*</td>
</tr>
<tr>
<td>log-alcohol use per grade</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{e_1,e_2/\text{PEER}}$ True initial status and rate of true</td>
<td>-.006</td>
</tr>
<tr>
<td>change: partial covariance of true log-alcohol use in grade seven and true</td>
<td></td>
</tr>
<tr>
<td>change in log-alcohol use per grade</td>
<td></td>
</tr>
<tr>
<td>Population true mean trajectory in peer alcohol use</td>
<td></td>
</tr>
<tr>
<td>$\mu_{\alpha_1}$ True initial status: average peer true log-alcohol use</td>
<td>.188***</td>
</tr>
<tr>
<td>in grade seven</td>
<td></td>
</tr>
<tr>
<td>$\mu_{\alpha_2}$ Rate of true change: average true change in peer log-</td>
<td>.096***</td>
</tr>
<tr>
<td>alcohol use per grade</td>
<td></td>
</tr>
<tr>
<td>Population true residual variances and covariance in peer alcohol use</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\alpha_1}^2$ True initial status: variance of true peer log-</td>
<td>.070***</td>
</tr>
<tr>
<td>alcohol use in grade seven</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\alpha_2}^2$ Rate of true change: variance of true change in</td>
<td>.029**</td>
</tr>
<tr>
<td>peer log-alcohol use per grade</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\alpha_1,\alpha_2}$ True initial status and rate of true</td>
<td>.001</td>
</tr>
<tr>
<td>change: covariance of true peer log-alcohol use in grade seven and true</td>
<td></td>
</tr>
<tr>
<td>change in peer log-alcohol use per grade</td>
<td></td>
</tr>
<tr>
<td>Population regression of change in alcohol use on change in peer alcohol</td>
<td></td>
</tr>
<tr>
<td>use $\gamma_{\alpha_1,\alpha_2}$ Regression of true adolescent log-</td>
<td>.799***</td>
</tr>
<tr>
<td>alcohol use in grade seven on true peer log-alcohol use in grade seven</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\alpha_2,\alpha_1}$ Regression of true adolescent log-alcohol</td>
<td>.081</td>
</tr>
<tr>
<td>use in grade seven on the rate of true change in peer log-alcohol use</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\alpha_1,\alpha_2}$ Regression of true rate of change in</td>
<td>-.143*</td>
</tr>
<tr>
<td>adolescent log-alcohol use on true peer log-alcohol use in grade seven</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{\alpha_2,\alpha_1}$ Regression of true rate of change in</td>
<td>.577**</td>
</tr>
<tr>
<td>adolescent log-alcohol use on the rate of true change in peer log-alcohol</td>
<td></td>
</tr>
<tr>
<td>use</td>
<td></td>
</tr>
</tbody>
</table>

$\sim = p < .10, ** = p < .01, *** = p < .001. \chi^2(4, 1122) = 11.54 (p = .021)$
IV. DISCUSSION

In this chapter, we have shown how individual growth modeling can be accommodated within the general framework of covariance structure analysis. We have explored links between these two formerly distinct conceptual arenas, laying out the mapping of one onto the other, and showing how the new approach provides a convenient way of addressing research questions about individual change. This innovative application of covariance structure analysis offers many flexible data-analytic opportunities.

First, the method can accommodate any number of waves of longitudinal data. Willett (1988, 1989) showed that the collection of more waves of data leads to higher precision for the estimation of individual growth trajectories and greater reliability for the measurement of change. In the covariance structure analyses of change, extra waves of data extend the length of the empirical growth record and expand the sample between-wave covariance matrix (thereby increasing degrees of freedom for model fitting), but do not change the fundamental parameterization of the level-1 and level-2 models.

Second, the occasions of measurement need not be equally spaced. Change data may be collected at irregular intervals either for convenience (e.g., at the beginning and end of the school year) or because the investigator wishes to estimate certain features of the trajectory more precisely by clustering data-collection points around times of greater research interest. Such irregularly spaced data is accommodated by the method, provided everyone in the sample is measured on the same set of irregularly spaced occasions within each domain. When that is not the case, the analyses still can be conducted using multigroup analysis.

Third, individual change can be either a straight line or curvilinear. The approach can accommodate not only polynomial growth of any order but also any type of curvilinear growth model in which status is linear in the individual growth parameters. In addition, because the goodness-of-fits of nested models can be compared directly under the covariance structure approach, one can systematically evaluate the adequacy of contrasting individual growth models in any empirical setting.

Fourth, the covariance structure of the occasion-by-occasion level-1 measurement errors can be modeled explicitly. The population measurement error covariance matrix is not restricted to a particular shape or pattern. The investigator need not accept unchecked the level-1 independence and homoscedasticity assumptions of classical analyses, nor the band-diagonal configuration required by repeated-measures analysis of variance.
Indeed, under the covariance structure approach, a variety of reasonable error structures can be systematically compared and the most appropriate structure adopted.

Fifth, the method of maximum likelihood provides overall goodness-of-fit statistics, parameter estimates, and asymptotic standard errors for each hypothesized model. By using the covariance structure method, the investigator benefits from the utility of a well-documented, popular, and well-understood statistical technique. Appropriate computer software is widely available. In this chapter, we have relied upon the LISREL computer package, but our techniques can easily be implemented using other software such as EQS (Bentler, 1985), LISCOMP (Muthén, 1987), and PROC CALIS (SAS, 1991).

Sixth, by comparing goodness-of-fit across nested models, the investigator can test complex hypotheses about interindividual differences in true change. One benefit of fitting an explicitly parameterized covariance structure to data using a software package like LISREL is that selected model parameters can be individually or jointly constrained during analysis to particular values. This allows the investigator to conduct tests on the variability of the individual growth parameters across people. We can, for instance, fix the value of one parameter to a value common across individuals but permit another parameter to be random.

Finally, the flexibility of the general LISREL model permits extension of the analysis of change in substantively interesting ways. For example, we can predict change in one or more domains by simultaneous changes in several other domains. Furthermore, we can introduce additional exogenous variables as predictors of any or all of these changes. The method also enables the modeling of intervening effects, whereby a predictor may not act directly on change, but indirectly via the influence of intervening factors, each of which may be either time-invariant or a measure of change itself.

REFERENCES


APPENDIX

Sample Mean Vectors and Covariance Matrices For the Adolescent Alcohol Use Example

<table>
<thead>
<tr>
<th>Variables</th>
<th>Means</th>
<th>Female</th>
<th>Alcohol use</th>
<th>Peer alcohol use</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FEM</td>
<td>Start of seventh grade</td>
<td>End of seventh grade</td>
</tr>
<tr>
<td>Untransformed data</td>
<td></td>
<td></td>
<td>ALC1</td>
<td>ALC2</td>
</tr>
<tr>
<td>FEM</td>
<td>0.612</td>
<td>0.238</td>
<td>0.516</td>
<td>0.651</td>
</tr>
<tr>
<td>ALC1</td>
<td>1.363</td>
<td>-0.019</td>
<td>0.304</td>
<td>0.335</td>
</tr>
<tr>
<td>ALC2</td>
<td>1.421</td>
<td>-0.034</td>
<td>0.255</td>
<td>0.323</td>
</tr>
<tr>
<td>ALC3</td>
<td>1.491</td>
<td>-0.012</td>
<td>0.255</td>
<td>0.323</td>
</tr>
<tr>
<td>PEER1</td>
<td>1.347</td>
<td>-0.018</td>
<td>0.304</td>
<td>0.200</td>
</tr>
<tr>
<td>PEER2</td>
<td>1.578</td>
<td>-0.051</td>
<td>0.284</td>
<td>0.470</td>
</tr>
<tr>
<td>PEER3</td>
<td>1.684</td>
<td>-0.053</td>
<td>0.255</td>
<td>0.323</td>
</tr>
</tbody>
</table>

Transformed data

<table>
<thead>
<tr>
<th>Variables</th>
<th>Means</th>
<th>Female</th>
<th>Alcohol use</th>
<th>Peer alcohol use</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FEM</td>
<td>Start of seventh grade</td>
<td>End of seventh grade</td>
</tr>
<tr>
<td>FEM</td>
<td>.612</td>
<td>.238</td>
<td>.136</td>
<td>.555</td>
</tr>
<tr>
<td>ALC1</td>
<td>.225</td>
<td>-.008</td>
<td>.136</td>
<td>.155</td>
</tr>
<tr>
<td>ALC2</td>
<td>.254</td>
<td>-.013</td>
<td>.078</td>
<td>.082</td>
</tr>
<tr>
<td>ALC3</td>
<td>.288</td>
<td>-.005</td>
<td>.065</td>
<td>.082</td>
</tr>
<tr>
<td>PEER1</td>
<td>.177</td>
<td>-.009</td>
<td>.066</td>
<td>.045</td>
</tr>
<tr>
<td>PEER2</td>
<td>.290</td>
<td>-.022</td>
<td>.064</td>
<td>.096</td>
</tr>
<tr>
<td>PEER3</td>
<td>.347</td>
<td>-.024</td>
<td>.060</td>
<td>.074</td>
</tr>
</tbody>
</table>