





TAS

This dimension is intended to capture teachers' errors in doing and talking about mathematics, which can occur when solving problems, defining terms, launching tasks, making points about mathematics, in the notation that is used, and so on.



- Codes in Errors and Imprecision:
  - Major mathematical errors
  - Imprecision in mathematical language or notation
  - Lack of clarity in presentation of mathematical content
  - Overall Errors and Imprecision





- Notes:
  - Designed to get at related things



High means "low quality"

- Quantity codes
- Even if an error comes from the curriculum materials, it is still recorded as an error





## **Major Mathematical Errors**

- Definition: You say "I can't believe that just happened" and "mathematically totally incorrect."
- Examples:

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- Solving a problem incorrectly (and not correcting it)
- For errors made by a student rather than by the teacher:
  - Major errors need to be corrected (eventually)
  - Language imprecision and lack of clarity made by the student are NOT considered when scoring those codes
- Misconstruing/misdefining a major mathematical concept
  - Calling 4 + (6 + 7) = (4 + 6) + 7 commutativity
- Evaluating a solution method as incorrect when it is correct
- Distinguish from:
  - Lack of clarity in that it is mathematically wrong and major
  - Imprecision in language or notation in that it takes up more "space" in the classroom; content not taught correctly





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## Imprecision in Language or Notation

• Components:

- Errors and imprecision in notation
- Errors and imprecision in mathematical language
- Errors and imprecision in general language
- Examples:
  - Errors in use of the equals sign
    - 4 + 1 = 5 + 2 = 7
  - Expression vs. equation; calling "perimeter" "parameter"; "timesing"
  - "Borrowing," "less than triangles," "reducing"
- Distinguish from:
  - Major errors in that these errors, while mathematically incorrect, are more fleeting; content as conveyed is not wholly inaccurate, although the language is
  - Lack of clarity in that the imprecision may be brief (one sentence); lack of clarity is usually longer





## Imprecision in Language or Notation

- Automatically record as an imprecision
  - "Reducing" fractions
  - Referring to "bigger" and "smaller" equivalent fractions
  - "You can't subtract a larger number from a smaller number"
    - Also for division
  - "Dividing makes numbers smaller"
  - Expressions vs. equations
  - Misuse of equals sign (=)



- Consider in context or in combination with other language
  - "Timesing," "minusing" (etc)
  - "Alligator mouth" for <>
  - "Top" and "bottom" for fractions



## Lack of Clarity

- Definition: You have to ask: "What, mathematically, was the teacher trying to say?"
- Examples:

1

- Discussion of why 7 + -3 = 4 heads toward "-4 is too small to be the answer"
  - This is not wrong, but the mathematical point is not clear.
- Teacher endorses conflicting definitions for same concept
- "The area is a number of square units needed to cover the figure, and we've talked before about the box like a gift that somebody gives you. The box itself and everything inside the box is the area, but the wrapping paper around it would be like surface area and we talked about that and we talked about the perimeter is walking around the fence around an area."





# Lack of Clarity

- Distinguish from:
  - Major mathematical errors:
    - Teacher is unable to explain mathematical ideas to students
    - Teacher utterances are not necessarily mathematically incorrect
  - Imprecision in language or notation:
    - Some overlap
    - Lack of clarity generally longer "space" in the class
- If a major mathematical error occurs:
  - Use your judgment about whether it makes the mathematics unclear for students





### Generally for Imprecision and Lack of Clarity

- There is some wiggle room!
- We're not measuring precision; we're measuring imprecision
- We're not measuring total clarity (100%), we're measuring *lack of clarity*
- Our standard is not 100% totally clear and precise
  - It cannot be, because no teacher is totally clear and precise
  - Standard: Is there enough imprecision/lack of clarity to obscure the mathematical point?





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## **Overall Errors and Imprecision**

- Always scored with a "2" if any error is identified
- Qualitative judgment:
  - No errors or imprecision (1)
  - Several minor errors or slips of the tongue; not enough to suggest teacher has problems with content, but enough that a mathematician would be seriously annoyed; somewhat muddled presentation (2)
  - Multiple small errors, one large error, or consistent lack of clarity (3)





## Examples (Score all 4 codes)

- Mimi: Greater than, Less than
- Karen: Cows and Calves
- Georgia: Equations and Estimation
- Georgia: Equivalent Fractions





**Guiding Questions:** 

- Does the teacher make a major or serious mathematical error when solving a problem, defining a term, or explaining a process?
- Does the teacher make errors in notation, mathematical language, or general language?
- Is part or all of the teacher's presentation or explanation unclear?





### Mimi: Greater than, Less than

• Third grade

1

 The teacher has just passed out two handouts, one with an equals sign and the other with a < sign.</li>





#### Mimi: Greater than, Less than: Video







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## How would you score this clip for:

- Major mathematical errors
- Imprecision in mathematical language or notation
- Lack of clarity in presentation of mathematical content
- Overall Errors and Imprecision
- Take a moment to write down your scores before moving on to our answers...





#### Mimi: Greater than, Less than: Answers

- Major mathematical errors: 3
  - she calls a less than sign the greater than sign
- Imprecision in language or notation: 3
  - "lesser than number," "higher number," having to have the same "thing" on both sides (she means same quantity or value)
- Lack of clarity: **3** 
  - not sure what she's talking about
- Overall: 3





### Karen: Cows and Calves

- On overhead, there are three groups with two cows in each group
- Students are making up number sentences about the legs of two cows and four calves drawn on the overhead (3 groups of cows and calves, total)





### Karen: Cows and Calves: Video







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- Major mathematical errors
- Imprecision in mathematical language or notation
- Lack of clarity in presentation of mathematical content
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### Karen: Cows and Calves: Answers

- Major mathematical errors: 2
  - brief 24 / 0 = 0
- Imprecision in language or notation: 1
  - no imprecision in language or notation
- Lack of clarity: 1
  - direction of the lesson is clear
- Overall: **2** 
  - momentary





### Georgia: Equations and Estimation

Sixth grade

1

- Special education class
- Working on estimating the value of the unknown in algebraic equations





#### Georgia: Equations and Estimation: Video







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## How would you score this clip for:

- Major mathematical errors
- Imprecision in mathematical language or notation
- Lack of clarity in presentation of mathematical content
- Overall Errors and Imprecision
- Take a moment to write down your scores before moving on to our answers...





#### Georgia: Equations and Estimation: Answers

- Major mathematical errors: 1
  - no major errors
- Imprecision in language or notation: 2
  - reference to average instead of estimate
- Lack of clarity: 2
  - the first 1:30 minutes of the segment were okay; they are discussing mental math and forming an equation to represent the baseball card problem
  - not clear what the goal of this activity was and how students were supposed to work on this task, e.g., "What will we change these numbers to?"
- Overall: **2**





### Georgia: Equivalent Fractions

• Sixth grade

Working on finding equivalent fractions





# Georgia: Equivalent Fractions: Video







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## How would you score this clip for:

- Major mathematical errors
- Imprecision in mathematical language or notation
- Lack of clarity in presentation of mathematical content
- Overall Errors and Imprecision
- Take a moment to write down your scores before moving on to our answers...





# Georgia: Equivalent Fractions: Answers

- Major mathematical errors: 1
  - no major error
- Imprecision in language or notation: 3
  - several: "reducing," "timesing," "making smaller," equals sign in GCF, etc
- Lack of clarity: 2
  - making "smaller and bigger equivalent fractions;" notice that the presentation of the procedure was very clear
- Overall: 3







# **Errors and Imprecision**

Please move on to the Errors and Imprecision practice module.



