Richness of the Mathematics

Part I

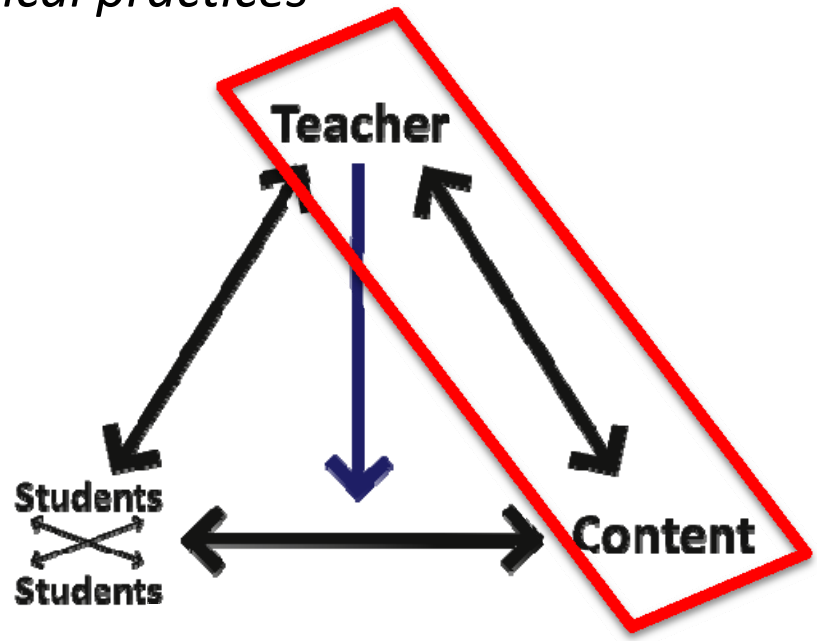
National Center for Teacher Effectiveness

Center for Education Policy Research
HARVARD UNIVERSITY

Richness of the Mathematics

This dimension captures the depth of the mathematics offered to students. Rich mathematics is focused on either:

- the *meaning* of facts and procedures; or
- *key mathematical practices*



Richness of the Mathematics

Codes in Richness of the Mathematics:

- **Meaning of facts and procedures:**
  - Linking and connections
  - Explanations
- **Mathematical practices:**
  - Multiple procedures or solution methods
  - Developing mathematical generalizations
  - Mathematical language
- **Overall richness of the mathematics**

Richness of the Mathematics

Guiding Questions:

- Does the segment convey a sense of *why* facts are true, procedures work, or problems have been solved in a particular way?
- Does the segment feature any mathematical practices, including examining and comparing solution methods, making mathematical generalizations, or using precise language?

Meaning-Oriented Codes

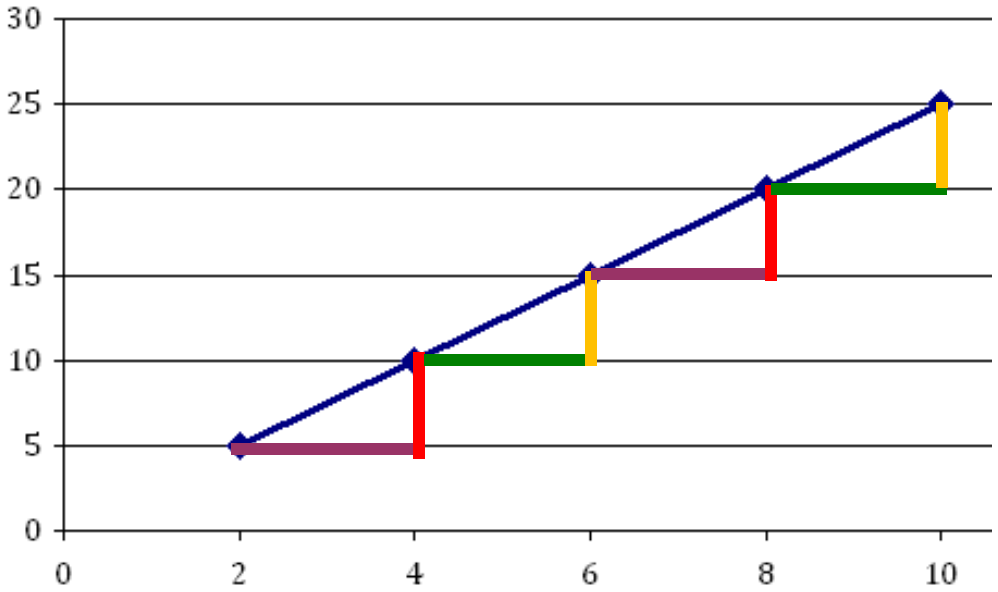
- Linking and Connections
- Explanations

Linking and Connections

- *Definition:* This code captures explicit links and connections that are drawn:
  - among different *mathematical ideas* or *procedures* OR
  - among different *representations* of mathematical ideas or procedures OR
  - across *representations* and *mathematical ideas* or *procedures*

Linking and Connections

X	Y
2	5
4	10
6	15
8	20
10	25



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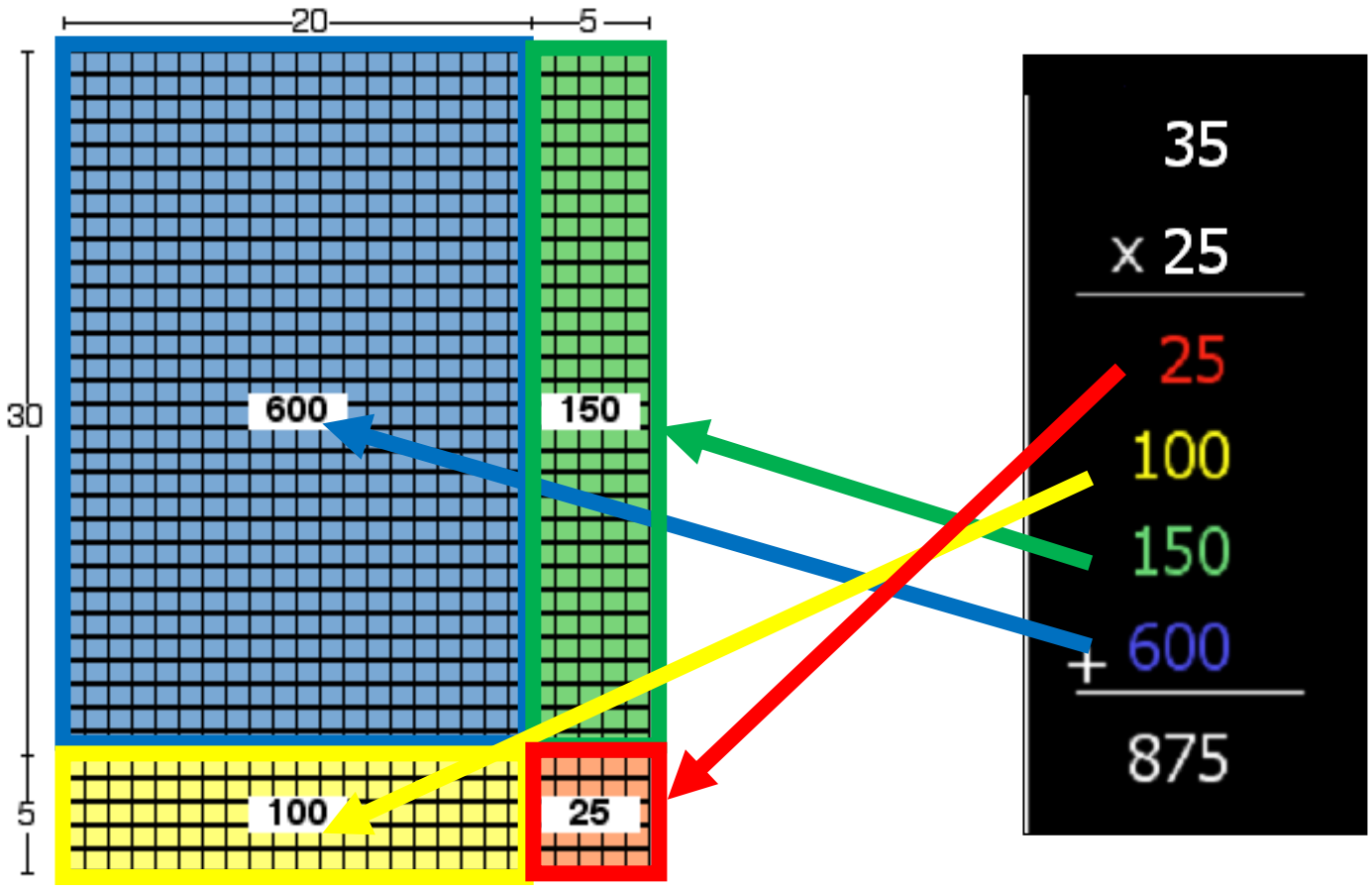
  - across *representations* and *mathematical ideas* or *procedures*

Linking and Connections

- **High (3)**
  - Links and connections are present with *sustained*, *careful* work characterized by one or more of the following features:
    - **Explicitness** about how two or more ideas, procedures, or representations are related (e.g., pointing to specific areas of correspondence)

Linking and Connections: Explicitness



Linking and Connections

- **High (3)**
  - Links and connections are present with *sustained*, *careful* work characterized by one or more of the following features:
    - **Explicitness** about how two or more ideas, procedures, or representations are related (e.g., pointing to specific areas of correspondence), OR
    - **Detail and elaboration** about how two mathematical ideas, procedures, or representations are related to one another (e.g., providing information about under what conditions the relationship occurs; noting meta-features; discussing implications of relationship)

Linking and Connections

- **Mid (2)**
  - Links and connections are present, but do not have the features included in high, OR
  - Linking and connections occur only momentarily
- **Low (1)**
  - No linking and connections occur, OR
  - *Incorrect* links or connections, OR
  - Linking and connections are *completely* pro forma (e.g., "Yesterday we added fractions with like denominators, today we will subtract fractions with like denominators.")

Explanations

- *Definition*: Explanations explain WHY a fact is true, a procedure works, or an answer to a problem is correct
- This includes giving mathematical meaning to ideas, solution methods, answers, steps in a procedure, etc.
- *Examples*:
  - You can simplify 4/8 by dividing both top and bottom by 4, which is the same as dividing by 4/4. Because dividing by 4/4 is the same as dividing by 1, this does not change the value.
  - To determine whether a number is divisible by 4, you can look at the last two digits. This is because every number over 100 can be represented as….

Explanations

- *Distinguish from*:
  - Statements that describe "how" rather than explain "why" ("to simplify 4/8, first I divided 4 by the 4, then I divided 8 by the 4, and got 1/2"). *Narrations of procedures do not count*.
  - Statements of fact or definitions without additional connections or linkages
    - "Rectangles have two pairs of equal sides and 4 right angles" *is not an explanation*
    - "All squares are rectangles" *is not an explanation*
    - "A square is a rectangle, because a square meets the definition of a rectangle: it has two pairs of equal sides and 4 right angles" *is an explanation*

Once it's an Explanation: Mid or High?

- **Low (1)**
  - No explanations
  - Incorrect explanations

- **Mid (2)**
  - *Local* explanations – of a specific problem
    - E.g., a student says "I simplified 4/8 by dividing both top and bottom by 4. That's the same as dividing by 4/4 which equals 1, and dividing by 1 doesn't change the value of 4/8."
  - Global explanations that are not complete or detailed

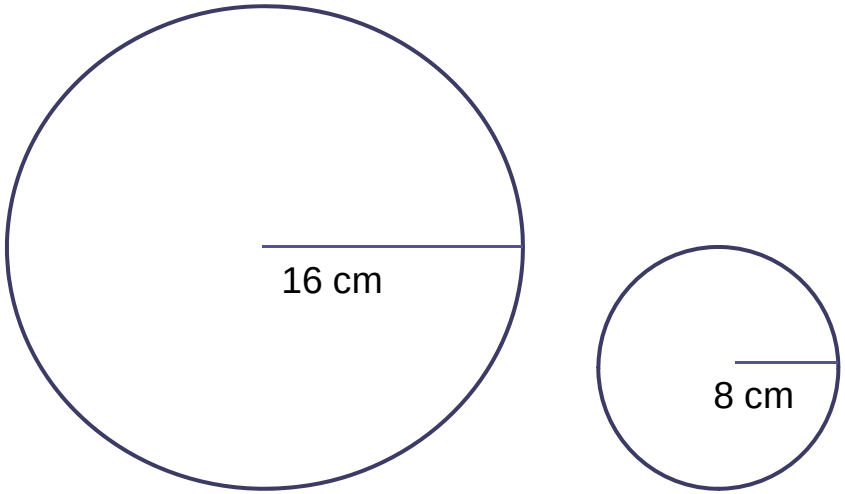
Once it's an Explanation: Mid or High?

- **High (3)**
  - High is used for a *global* mathematical explanation that generalizes past specific problems
    - E.g., a student says "I simplified 4/8 by dividing both top and bottom by 4, which is the same as dividing by 4/4. Any time you divide by a fraction that is equivalent to 1, the value of the initial fraction does not change."
  - AND meets *one or more* of the following criteria:
    - They give meaning to the mathematics under study
    - They are detailed

Note: The document "Examples of Local and Global Explanations.docx" has additional examples and can be found under the attachments link in the top right corner of the player.

Local vs. Global Explanations

- You and your friends are really hungry. You can either order two medium pizzas or one large pizza. What would you prefer?



Local vs. Global Explanations

- Local Explanation: “Definitely the larger pizza. We know that the area of a circle is π times the square of the radius. So, the large pizza has an area of 16 × 16 × π, which is about 804 square centimeters. The medium one has an area of 8 × 8 × π which is about 201 square centimeters. So two mediums are smaller than a large. Definitely the larger pizza.”

- Global Explanation: “Definitely the larger pizza. We know that the area of a circle is π times the square of the radius. *Because the radius of the larger pizza is twice as big as that of the medium pizza,* the larger pizza is four times bigger than the medium pizza.”

General Notes on Meaning-Oriented Richness Codes

- Rate segments as low when they contain substantially incorrect elements of richness:
  - Unclear or incomplete explanations
  - Incorrect or inappropriate links between ideas
- During student work time and/or for student statements
- These are quality codes – you can assign a rating of high even if that aspect of instruction occurs for only a portion of the segment.

Meaning-Oriented Codes: Examples (Score for two codes)

- Karen: Long Division
- Lauren: Likelihood Line
- Bianca: Integer Subtraction
- Karen: Interpreting Remainders
- Lisa: Inverse Operations

Karen: Long Division

- 3rd grade
- Teacher has split the class into two groups; one group is working more or less quietly on another activity
- The other group has gathered at her feet to review division

Karen: Long Division: Video

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How would you score this clip for:

- Linking and Connections
- Explanations

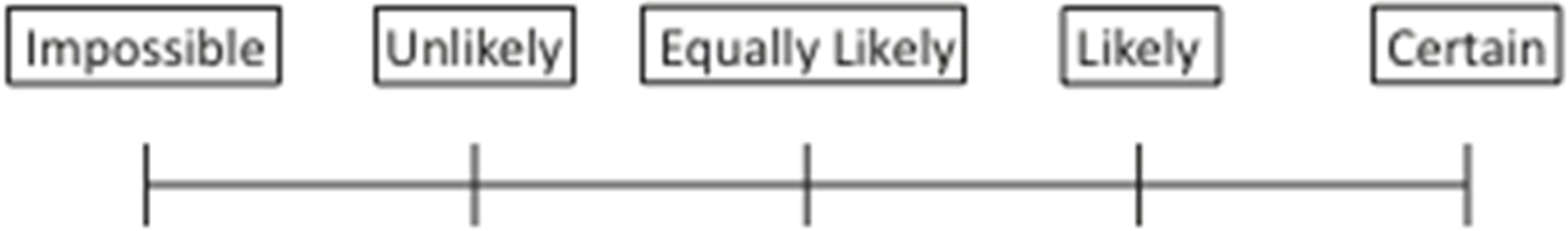
- *Take a moment to write down your scores before moving on to our answers…*

Karen: Long Division: Answers

- Linking and connections: **3**
  - Sustained and careful linking of:
    - cubes (representation) and dividing whole numbers (a mathematical procedure)
    - numerical symbols and cubes (linking representations)
- Explanations: **2**
  - Meaning of division for 72÷4 (local explanation)

Lauren: Likelihood Line

- 4th grade
- Lesson on probability
- The class has been talking about probability for *dichotomous* events



Lauren: Likelihood Line: Video

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Lauren: Likelihood Line: Answers

- Linking and connections: **3**
  - The whole clip is about the connections between the mathematical ideas of notions of likelihood and the ordering of numbers between 0 and 1 ("the way mathematicians would name those points"). She is explicit in the connections between these (e.g., 1 , 100%, "for sure", "it's definitely going to happen")
  - Also links fractions and percents, but this aspect of instruction was not sustained enough to merit a 3 in this code on its own
- Explanations: **3**
  - Global explanations offered by students:
    - The meaning of 100% is "for sure"; "definite"; "It will happen"
    - "Like fifty is half cause a hundred percent and a hundred percent is full. So fifty would be half of a hundred so I'm thinking like fifty-fifty means like each half is equal, so I would put half."

Bianca: Integer Subtraction

- 7th grade
- The class uses blue (positive) and red (negative) chips to represent integers
- The class then uses this chip model to solve 5 – -7

Bianca: Integer Subtraction: Video

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Bianca: Integer Subtraction: Answers

- Linking and Connections (**1**)
  - No connections:
    - Teacher solves the problem 5 – (-7) with the chips, but she stays "in the chips," never connecting the moves she is making to the expression: what is 5, what is -7, connecting subtraction to "taking away" chips
    - Take-away model: chips corresponding to the subtrahend should not be represented on the overhead.
- Explanations (**1**)
  - No explanation offered

Karen: Interpreting Remainders

- 5th grade
- Class is working on different interpretations of remainders

Karen: Interpreting Remainders: Video

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How would you score this clip for:

- Linking and Connections
- Explanations

- *Take a moment to write down your scores before moving on to our answers…*

Karen: Interpreting Remainders: Answers

- Linking and Connections: (**1**)
  - No links or connections are actually made (remainders and divisors both pertain to division so she is not linking *different* mathematical ideas)
- Explanations: (**3**)
  - Explains what a remainder is *generally;* she is not talking about a specific problem
  - She is complete and clear in her explanation

Lisa: Inverse Operations

- 5th grade
- Class was working on solving algebraic equations using "inverse operations"

$$n - 2 = 7 \rightarrow n = 2 + 7$$

- However, in some cases this approach did not seem to work

$$31 - n = 12 \not\rightarrow n = 31 + 12$$

$$n \times 2 = 14 \not\rightarrow n = 2 \div 14$$

- The teacher explains why the inverse operation approach can be used in some cases but not in others

Lisa: Inverse Operations: Video

How would you score this clip for:

- Linking and Connections
- Explanations

- *Take a moment to write down your scores before moving on to our answers…*

Lisa: Inverse Operations: Answers

- Linking and connections (**1**)
  - No connections are made (the teacher just states that subtraction and division are not commutative)
- Explanations (**1**)
  - The teacher provides an "explanation"
  - AND the teacher is correct in noting that subtraction and division are not commutative
  - BUT this does not explain why the inverse operations cannot be used when the variable appears in the second place

End of Richness Part I

Please move on to Richness Part II

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