





This dimension captures whether teachers can understand and respond to students' mathematically substantive productions (utterances or written work) or mathematical errors.



- Codes in Working with Students and Mathematics:
  - Remediating student errors and difficulties
  - Responding to student mathematical productions in instruction
  - Overall Working with Students and Mathematics





Guiding Questions:

- Do students make errors or encounter difficulty with content?
  - Do teachers provide remediation of student errors? If so, of what kind?
- Do students contribute spoken or written ideas, or productions, to the lesson?
  - Can teachers understand these productions?
  - Can teachers use those productions in the course of instruction?





• *Definition*: With this code, we mean to mark instances of remediation in which student misconceptions and difficulties with the content are *substantially* addressed. There are two main teacher actions that fall into this code: *extended procedural remediation* and *conceptual remediation*.

• Examples:

- Conceptual remediation:
  - Identifying/addressing the source of student errors or misconceptions: "I noticed that some of you forgot to multiply both sides of the equation by x. What happens if you multiply one side by x and not the other?....(more)"
  - Pointing to underlying meaning when responding to errors: "You cannot simply add a ten here. Remember, the number we start with needs to remain the same, regardless of how it is represented..."





- Examples of <u>extended procedural</u> remediation
  - Whole-group work to correct student error(s) in a procedure.
    These must be of reasonable length, and cover key portions of the procedure in depth
  - Correcting student work individually. An extended procedural remediation with one or more students. Work with at least one student must be of reasonable length and cover key portions of the procedure in depth.





- Other instances of remediation:
  - Anticipating student errors: "Remember to multiply both sides of the equation by x; every year I get students who forget the second side."
  - Parsing student statements: "What Andrew said is partially correct and partially incorrect. He was correct in noting....."





- Distinguish from:
  - "Routine" remediation e.g., "15 is not correct. What is the correct answer?"
  - Brief procedural remediation e.g. "You skipped a step" or "To find the slope of this perpendicular line, you take the opposite of the original slope and then find its reciprocal."
  - Cases in which teacher's remediation is incorrect (i.e., teacher's intervention leads students to an incorrect mathematical path or conclusion)
- Note: Remediation may occur either during active instruction or during individual/small group work time





- Low (1): No conceptual or extended procedural remediation, for any of the following reasons:
  - There are no student misunderstandings or difficulties with the content
  - Remediation is non-mathematical
  - Remediation is purely procedural and brief
  - There is a student error but the teacher chooses not to remediate
  - The teacher remediation is confusing or off-track





• Mid (2):

- Teacher engages in *conceptual* remediation *briefly* or *occasionally*.
- Also includes *extensive* procedural remediation.





- High (3):
  - *Extended conceptual* remediation characterized by:
    - Identifying the source of student errors or misconceptions
    - Discussing how student errors illustrate broader misunderstanding and then addressing those errors.
  - Other evidence of high-quality remediation: Even a brief instance of one of the following teacher behaviors is rated as high for remediation:
    - anticipating common student errors and providing instruction that helps avoid error
    - parsing student productions to separate correct and incorrect thinking.





## Remediation of Student

# Errors and Difficulties : General Notes

- Giving a review/synopsis to a student who was absent is not counted as remediation.
- Helping students with use of mathematical tools (e.g. compass, calculator) counts as remediation only if it is mathematical in nature. E.g.:
  - Remediation: "You have to keep the compass at the same setting as you draw so that your circle has the same radius everywhere."
  - Not remediation: "You've got the window, now you need to change the zoom settings on your calculator."
- Even a brief instance of anticipating common student errors should be coded as high





# Mathematical Productions in Instruction

- Definition of student "productions": Substantive student contributions that evoke mathematical ideas for the class.
  - Productions must have some mathematical substance (e.g., questions, solutions, explanations, generalizations that *contribute to the development* of the math topic under discussion).
  - Productions will tend to have features of explanation or generalization, be a "why" question, or be a complex description of a solution method.
  - Productions need not be mathematically correct.
  - Productions are not simply answers to questions where teacher has sought a specific, bounded piece of information.
  - If a student's utterance is inaudible, do not count it as a production, unless there is good evidence to infer what the student said (e.g. teacher revoices the idea in student language).





# Mathematical Productions in Instruction

- Examples of what student comments qualify as mathematical productions: (Usually depends on context):
  - "I've noticed a pattern in the numbers.....So, does this pattern mean that 0 is an even number?"
  - "Can you have a parabola that goes sideways?"
  - "If the space between two dots on the grid paper is 1 unit, then each side of this triangle 
    is 1 unit long, and the diagonal is also 1 unit long."
- *Non-examples*: Typically (but not necessarily) short:
  - "The answer is 3."
  - "We need to invert the second fraction and then multiply."
  - "I am not getting this."



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## Mathematical Productions in Instruction

#### Guiding questions

- Were there student productions?
- If so, were they taken up by the teacher?
- Low (1):
  - No student productions
  - Student productions present but the teacher:
    - Ignores those productions
    - Does not understand those productions
    - Uses productions in unproductive or confusing ways





Mathematical Productions in Instruction

• Mid (2):

- Brief use of some of the "high" features
- Evidence that teacher understands student productions but chooses not to use them





# Mathematical Productions in Instruction

• High (3):

- Teacher "hears" what students are saying mathematically, and responds substantively during instruction *and*
- Students' productions are *substantially* woven into the *development* of mathematical ideas during the lesson
- Teacher may elicit student ideas or solutions, ask other students to comment on these, expand on and reinforce student utterances
- Examples:
  - Identifying key ideas: "Alex noticed that ..."
  - Pointing to the mathematical value/essence of a production: "Michelle's question is important because..."
  - Inviting other students to reflect on another student's contribution: "You argued that all the triangles have at least one acute angle. What do other people think about this conjecture?"



## Mathematical Productions: General Notes

- Not every student utterance counts we care about mathematically substantive productions
- Not every remediation counts we care about only extended procedural or conceptual remediation
- A teacher associating a student's name with an idea (e.g. "like Maya said before...") is often a trigger for "responding to student productions."
  - But of course, not every instance counts. It must be a *student* production and the teacher must use it at least briefly.





*Definition:* Overall evaluation of the teacher interactions with students and the mathematical content

• Low (1):

- Instruction is routine -- no opportunity for segment to be rated as "mid" or "high." Student errors and productions do not occur, OR
- Substantive student mathematical productions or errors do occur, but teacher does not respond to or use them , OR
- Student errors and productions may occur but teacher addresses them briefly/procedurally, OR
- Extensive procedural remediation, but not enough to dominate the segment, OR
- Teacher responses lead the lesson off-track, or are incorrect or inappropriate





• Mid (2):

- Teacher understands substantive productions *or* student errors, but the use of these is, cumulatively, brief or contains features of low and high.
- Extensive procedural remediation that dominates the segment or if the remediation is explicit and thorough





#### **High (3)**:

Teacher understands substantive student productions and responds to them in mathematically productive ways

#### OR

- A substantial conceptual remediation of student misunderstandings occurs. The remediation directly addresses, anticipates, or interprets student misunderstanding.
- Notes:
  - A lesson segment can be rated as High if it contains examples of *both* remediation and responding to student productions. At least one of these should be rated as high.
  - It can also be rated as High if it contains an *outstanding* example in only one of the two ٠ categories.
    - In the absence of one code, only stellar examples of the other code would bump the overall to a high.
  - A segment with only remediation cannot receive a High Overall rating unless the remediation is conceptual and at length.





• Distinguish from:

- How well the teacher presents the content
  - Captured by Overall Richness
- How students engage with the content
  - Captured by Overall Student Participation in Meaning-Making and Reasoning
- Other Notes
  - Teacher responding to student utterances that do not count as mathematical productions or errors
  - Generally treating students respectfully, having good rapport with students (these not captured within MQI instrument)





# Examples (Score for all 3 codes)

- Noel: Right angles in a cube
- Mercedes: Integer subtraction
- Marcus: The gum-ball problem
- Monique: Building bridges
- Mauricio: Square spinner
- Mauricio: Independent events





# Noel: Right Angles

• 3<sup>rd</sup> grade

- In a previous lesson, the class discussed different cube attributes
- In this lesson, a student makes a conjecture: "a cube has 24 right angles"





## Noel: Right Angles: Video







# How would you score this clip for:

- Remediating student errors and difficulties
- Responding to student mathematical productions in instruction
- Overall Working with Students and Mathematics
- Take a moment to write down your scores before moving on to our answers...





# Noel: Right Angles: Answers

- Remediating student errors and difficulties: 3
  - The teacher's drawing on the board at the end of the clip; remediation is conceptual (what does it mean to talk about right angles in a cube?) and extended
- Responding to student mathematical productions in instruction: 3
  - "You thought about it..."; also the whole clip pertained to addressing a student conjecture
- Overall: 3





## **Mercedes: Integer Subtraction**

- 7<sup>th</sup> grade
- Connected Mathematics
- Teacher has modeled how to use colored cubes to perform integer operations
- At this point, students are working in groups and the teacher circulates and supports them





#### Mercedes: Integer Subtraction: Video







# How would you score this clip for:

- Remediating student errors and difficulties
- Responding to student mathematical productions in instruction
- Overall Working with Students and Mathematics
- Take a moment to write down your scores before moving on to our answers...





## Mercedes: Integer Subtraction: Answers

- Remediating student errors and difficulties: 3
  - idea of opposites and also explaining adding pairs of chips that equal zero, and also; remediation is conceptual and extended
- Responding to student mathematical productions in instruction: 2
  - "what he is saying..."
- Overall: 3





#### Marcus: Gum-Ball Problem

- 6<sup>th</sup> grade
- Going over various probability problems
- At this point, the class considers the gum-ball problem: "If there are 36 gum balls in the machine, how many are purple? How many are yellow? How many are orange?"
- The class knows that 1/12 are purple, 1/6 are yellow, 3/4 are orange.





#### Marcus: Gum-Ball Problem: Video







# How would you score this clip for:

- Remediating student errors and difficulties
- Responding to student mathematical productions in instruction
- Overall Working with Students and Mathematics
- Take a moment to write down your scores before moving on to our answers...





## Marcus: Gum-Ball Problem: Answers

- Remediating student errors and difficulties: 1
  - A student says "I'm stuck" at the beginning, but there's no evidence teacher understands that student error. He just shows how.
- Responding to student mathematical productions in instruction: 2
  - largely ignores Nolan's correct solution method and presents a different approach, but does engage student with subtraction idea toward end
- Overall: 1
  - because there is no remediation and responding is weak





# Monique: Building Bridges

- 8<sup>th</sup> grade
- Connected Mathematics
- Class is conducting experiments with bridges (e.g., testing how strength of the bridge varies)
- The class is about to move into a new bridge experiment
- A student reads the instructions: set up a bridge, put a penny on it. Keep adding pennies until it breaks.





# Monique: Building Bridges: Video







# How would you score this clip for:

- Remediating student errors and difficulties
- Responding to student mathematical productions in instruction
- Overall Working with Students and Mathematics
- Take a moment to write down your scores before moving on to our answers...





# Monique: Building Bridges: Answers

- Remediating student errors and difficulties: 2
  - Teacher asks for commentary on Alex's method; re-voices correct conclusion; and it was a concept, but it was brief
- Responding to student mathematical productions in instruction: 2
  - "Alex said ... What do you guys think?" but brief
- Overall: **2** 
  - teacher seems comfortable with student thinking, uses
    Alex's idea; but both remediation and responding are brief





## Mauricio: Square Spinner

• 7<sup>th</sup> grade

- Connected Mathematics
- Probability tasks with respect to the following square spinner







#### Mauricio: Square Spinner: Video







# How would you score this clip for:

- Remediating student errors and difficulties
- Responding to student mathematical productions in instruction
- Overall Working with Students and Mathematics
- Take a moment to write down your scores before moving on to our answers...





## Mauricio: Square Spinner: Answers

- Remediating student errors and difficulties: 1
- Responding to student mathematical productions: 2
  - Refers to Angelo's idea to further explain Sammy's answer
- Overall: **2** 
  - Because of response to Angelo's idea







# Mauricio: Independent Events

- Same lesson
- The class has just discussed the probability of getting each of the numbers (or combinations thereof) in the following spinner, when spinning it once



 Now, they launch into reviewing the probability of getting a "one," if the spin the spinner twice





#### Mauricio: Independent Events: Video







# How would you score this clip for:

- Remediating student errors and difficulties
- Responding to student mathematical productions in instruction
- Overall Working with Students and Mathematics
- Take a moment to write down your scores before moving on to our answers...





## Mauricio: Independent Events: Answers

- Remediating student errors and difficulties: 3
  - the remediation is both conceptual and extended: teacher elicits student thinking, asks questions to clarify ideas, provides a counter-example to challenge a student's misconception: "Should I expect to get 10 one's in a row ¼ of a time?"
- Responding to student mathematical productions in instruction: 3
  - "What I hear him say is that each spin is independent of the other " and Chai's idea
- Overall: 3





Please move on to the Working with Students and Mathematics practice module.



