Using Schema-based Instruction to Improve Seventh Grade Students' Learning of Ratio and Proportion

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Paper presented at the annual meeting of the

American Educational Research Association, New York City, New York

Symposium 53.026: Research on Rational Numbers, Ratio, and Proportion

Hilton New York, Lincoln Suite, 4th Floor

Thursday, March 27, 2008

10:35 am to 12:05 pm

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Improving students' mathematical word problem solving skills has proved to be a significant challenge in the US. When solving story problems, children need to understand the language and factual information in the problem, translate the problem using relevant information to create an adequate mental representation, devise and monitor a solution plan, and execute adequate procedural calculations (Desoete, Roeyers, & De Clercq, 2003; Mayer, 1999). In short, solving word problems is closely related to *comprehension* of the relations and goals in the problem (e.g., Briars & Larkin, 1984; Cummins, et al., 1988; De Corte, Verschaffel, & De Win, 1985; Kintsch & Greeno, 1985; Riley, Greeno, & Heller, 1983). Despite their difficulty, story problems are critical in helping children connect different meanings, interpretations, and relationships to the mathematical operations (Van de Walle, 2007).

Although there is evidence to support reform based mathematics methods and curricula in enhancing the mathematics performance of high and average achieving students (Cohen & Hill, 2001; Fuson, Carroll, & Drueck, 2000; Schoenfeld, 2002), research on how to address low achieving students' difficulties with word problem solving is somewhat conflicting. In the field of mathematics education, the approach advocated by the NCTM Standards (and that is used in most of the NSF-funded reform curricula) advocates a student-centered, guided discovery approach for teaching students problem solving (Mayer, 2004; NRC, 2001). However, recommendations for this kind of instructional approach are at odds with the literature on problem solving instruction for low achieving students in the field of special education, which has found that low achieving students benefit far more from direct instruction and practice at problem solving than competent problem solvers (Baker, Gersten, & Lee., 2002; Jitendra & Xin, 1997; Tuovinen & Sweller, 1999; Xin & Jitendra, 1999). In fact, research conducted in reform-oriented classrooms indicates that many low achieving students (particularly those with learning disabilities) may assume passive roles and may encounter difficulties with the cognitive load of the discovery-oriented activities and curricular materials (Baxter, Woodward, & Olson, 2001; Baxter, Woodward, Voorhies, & Wong, 2002; Woodward, Baxter, & Robinson, 1999). Yet despite the clear supporting literature from special education in support of more direct instruction for low achieving students and those with learning disabilities, many in the mathematics education community have strong negative reactions to this instructional approach, in part because of perceived associations between direct instruction and the development of rote, inflexible knowledge.

The goal of the present study, which is a collaboration between a mathematics education researcher and a special education mathematics education researcher, was to design an instructional intervention specifically geared toward low achieving students which attempts to use the research literatures from both special education and mathematics education. Although we use a form of direct instruction, in keeping with the strong supporting literature on the success of such approaches for at-risk populations, we make two critical improvements to the ways that direct instruction has sometimes been (mis)applied.

First, it is clear that not all methods of direct instruction on problem solving strategies are equally effective with low achieving students and those with learning disabilities (Montague, Applegate, & Marquard, 1993). One weakness of many texts that adopt a direct instruction approach is that these texts are organized in a way that the same procedure (e.g., multiplication) is used to solve all problems on a page. As such, students do not have the opportunity to discriminate among different types of problems and approaches, perhaps leading to exclusive reliance (and perhaps rote memorization) on a small set of problem solving strategies. A second weakness is that conventional direct problem solving instruction teaches students to use key words (e.g., *in all* suggests addition, *left* suggests subtraction, *share* suggest division, Lester, Garofalo, & Kroll, 1989) and mechanical procedures (e.g., “cross multiply”) that do not develop conceptual understanding. These key word approaches ignore the meaning and structure of the problem and fail to develop reasoning and making sense of story situations (Van de Walle, 2007).

An approach to teaching word problem solving that relies on direct instruction but addresses the two weaknesses identified above emphasizes the role of the mathematical structure of problems. From schema theory, it appears that cognizance of the role of the mathematical structure (semantic structure) of a problem is critical to successful problem solution (Sweller, Chandler, Tierney, & Cooper, 1990). Schemas are domain or context specific knowledge structures that organize knowledge and help the learner categorize various problem types to determine the most appropriate actions needed to solve the problem (Chen, 1999; Sweller et al., 1990). For example, organizing problems on the basis of structural features (e.g., *rate* problem, *compare* problem) rather than surface features (i.e., the problem’s cover story) can evoke the appropriate solution strategy.

There is growing evidence regarding the benefits of schema training that focuses on priming the problem structure (Quilici & Mayer, 1996; Tookey, 1994; Chen, 1999; Fuchs, Fuchs, Prentice et al., 2003; Fuchs, Fuchs, Finelli, Courey, & Hamlet, 2004), including several published studies from our research team (Jitendra, Griffin, McGoey, Gardill, Bhat, & Riley, 1998; Jitendra, Griffin, Deatline-Buchman, & Sczesniak, 2007; Jitendra, Griffin, Haria, Leh, Adams, & Kaduvetoor, 2007; Griffin & Jitendra, in press). The goal of the present study is to extend our prior work and that of others by exploring the impact of explicit (direct) instruction in modeling problem solving strategies using visual (schematic) representations to highlight problem structure in the domain of ratio and proportion. We refer to this instructional model as schema-based instruction with self-monitoring, or SBI-SM.

The use of schematic representations is a particularly useful way to highlight underlying problem structure. A schematic diagram is a representation of the spatial relationships between parts of an object and spatial transformations (Hegarty & Kozhevnikov, 1999; Janvier, 1987; Sweller et al., 1990; Willis & Fuson, 1988). It is important to note that a schematic diagram is not merely a pictorial representation of the problem storyline but rather shows critical elements of the problem structure.

Our instructional use of schematic representations involves two types of processes: problem representation and problem solution (Harel, Behr, Post, & Lesh, 1992; Marshall, 1990; Mayer, 1992; Riley et al., 1983). The first or *comprehension phase* (problem schemata) involves translating the "definitive characteristics, features, and facts" (Marshall, 1990, p. 158) in the problem to construct a coherent representation of the problem. As such, instruction makes use of linguistic knowledge (e.g., gallons is the plural of gallon) and factual knowledge (e.g., there are 100 cents in a dollar) to determine the meaning of statements in the problems and teach students to identify the problem type (e.g., proportion), discern relevant and irrelevant information, and integrate the information in the problem into a coherent representation using schematic diagrams (Mayer, 1999). The second phase of problem solving, the *solution phase,* requires devising a plan (i.e., action schemata) to solve the problem and executing the plan (e.g., strategic knowledge). This phase involves (a) determining the sequence of steps, (b) selecting the operation (e.g., multiplication, division), (c) setting up the equation, and (d) executing the solution procedures (e.g., multiplying, dividing).

In addition, our instructional model explicitly fosters the use of student "think-alouds" to help in the development of metacognitive or self-monitoring skills. Teachers model how and when to use each problem solving strategy (Roehler & Cantlon, 1997) and work with students to reflect on the problem before solving it. For example, students read the problem aloud and try to understand what problem type it entails. When a new problem type is introduced, teacher questions prompt students to focus on both similarities and differences between the new problem and previously learned problem. In addition, students are prompted to describe *what* strategy can be used for solving the given problem and *why* it is appropriate as well as *how* to organize the information to solve the problem. Finally, students reflect on their understanding of the solution process by asking questions (e.g., "Does it make sense? How can I verify the solution?").

Method

Participants

Seventh grade students from eight classrooms and their teachers in a public, urban school participated in the study. For mathematics instruction, students in the school were grouped into same ability classes on the basis of their grades in mathematics from the previous school year (i.e., sixth grade). The school's classrooms represented four different ability levels: Honors (advanced), Academic (high), Applied (average), and Essential (low). In the present study, each treatment group (SBI-SM and control) included two sections of average and one each of high and low ability classrooms to adequately represent the high, average, and low ability levels in the school. Note that students in the Honors classrooms were not included in the study, because these students were learning advanced content (i.e., 8th grade). The sample of 148 students (79 girls, 69 boys) included those who were present for both the pretest and posttest.

The mean chronological age of students was 153.12 months (*range* = 137.04 to 174.96; SD = 5.76). The sample was primarily Caucasian (54%), and minority students comprised 22% Hispanic, 22% African American, and 3% American Indian and Asian. Approximately 42% of students received free or subsidized lunch and 3% were English language learners. Of the 15 (10%) special education students in the sample, 14 had an individualized education plan (IEP) in both mathematics and reading, whereas one student had an IEP in reading only. Further, five students (3%) in the study were English language learners (ELL).

Table 1 provides student demographic data by condition for each ability level status. A one-way analysis of variance (ANOVA) indicated that the difference between the two groups, F (1,146) = 0.36, $p = 0.55$, on age was not statistically significant. Chi-square analyses revealed no statistically significant between-group differences on gender, X^2 *(1)* = 0.04, p = *0.83*; ethnicity (i.e., white or non-white), X^2 *(2)* = 0.51, p = *048*; free or subsidized lunch, X^2 *(1)* = 0.12, p = *0.91*, and special education status, X^2 *(1)* = 2.75, p = *0.10*. The 148 children in the complete data set were demographically comparable to students who were absent on one or more days on which the pretest or posttest were administered.

All six teachers at the participating school were responsible for teaching mathematics in the different ability level classrooms. For the purpose of this study, two teachers each served as intervention or control teachers only. To control for teacher effects, the other two teachers served as both intervention and control teachers and taught students in the average ability classrooms. The teachers (3 females and 3 males) were all Caucasian, with a mean of 8.58 years of experience teaching mathematics (range 2 to 28 years). Three of the teachers held secondary education certification, four had a master's degree, and only three had a degree in mathematics. In these classrooms, the teachers used the Glencoe Mathematics: Applications and Concepts, Course 2 textbook (Bailey, Day, Frey, Howard, Hutchens, McClain, Moore-Harris, Ott, Pelfrey, Price, Vielhaber, & Willard, 2004) published by McGraw-Hill. In previous lessons, the teachers had covered integers and algebra, data analysis, and geometry.

Design

A pretest-intervention-posttest-retention test design was used to determine the potential efficacy of the SBI-SM intervention for improving seventh grade students' ratio and proportion problem solving skills. Classrooms were randomly assigned to either the intervention (SBI-SM) or control condition.

Intervention Materials

The SBI-SM intervention unit content provided the basis for solving problems involving ratios and proportions (see scope and sequence in Table 2). We identified specific concepts and problem-solving skills by reviewing the textbook used in the 7th grade classrooms and appropriately mapping the relevant topics to the ratio and proportion unit. This 10 daily 40-minute lessons unit included exercises to build an understanding of the concept of ratios and rates that are critical to understanding proportions and for engaging in proportional reasoning as well as to solving ratio and proportion word problems. Lessons 1 and 2 focused on the meaning of ratios, equivalent ratios, and comparison of ratios. Activities emphasized ratios in different contexts and included selecting an equivalent ratio and comparing ratios to identify ratios that are equivalent. Lessons 3 and 4 focused on expressing ratios in different ways (e.g., a:b; a is to b; a to b; a per b; for every b there is an a; n times as much/many as; nth of; a out of b); part-to-part ratios as one part of a whole to another part of the same whole (e.g., the ratio of girls to boys in the class); and part-to-whole ratios as comparisons of a part to a whole (e.g., the ratio of girls to all students in the class). Further, these lessons targeted solving ratio word problems that included the use of schematic diagrams and different solution strategies. A ratio problem involved comparing two quantities (part-part or part-whole) and expressing the multiplicative relationship between the quantities as a ratio. It involves a compared quantity, a referent quantity, and a ratio value. The ratio value describes the multiplicative relationship between the compared and the referent.

Lesson 5 extended the understanding of ratios to different quantities that are not parts of the same whole. Unlike a ratio that compares quantities with the same unit (e.g., 50 people out of 150 surveyed preferred Cherry Coke), a rate was defined as a comparison of two quantities with different units (e.g., inches to yards, miles to gallons, computers to students). Lessons 6 and 7 involved the application of rates in different contexts to solve proportion word problems. A proportion is a statement of equality between two ratios/rates (Van de Walle, 2007, p. 354). Often, there is an If-Then statement (proportional) that tells about two ratios/rates that are equivalent. Lessons 8 and 9 introduced scale drawings and scale models and required solving scale drawing problems (proportion). Lesson 10 examined the special way that fractions and percents are related. The lesson demonstrated how to express a quantity using either fractions or percents. Fractions and percents are different ways of expressing ratios – they both compare a part to the whole. For example, a ratio that compares a number to 100 is called a percent. In addition, the unit included homework problems to review the content presented in the daily lessons. Examples of items included in the materials were based on items derived from several published curricula (e.g., Glencoe, Prentice Hall, Scott Foresman, Everyday Math, Connected Math).

Measures

Mathematical problem-solving test (PS). To assess mathematics competence on ratio and proportion problems, students completed a researcher-designed mathematical PS test prior to instruction (pretest), immediately following instruction (posttest), and four months following instruction (delayed posttest). The PS test consisted of 18 items derived from the TIMSS, NAEP, and state assessments and assessed ratio and proportion problem solving knowledge similar to the instructed content (see sample item in Figure 1). Students had 40 minutes to complete the same 18-item test at pretest, posttest, and delayed posttest. Directions for administering the problem-solving test required students to show their complete work. Scoring involved assigning one point for the correct answer and "0" for an incorrect answer. On this sample, Cronbach's alpha was 0.73 for the pretest, 0.78 for the posttest, and 0.83 for the delayed posttest.

Treatment implementation fidelity. Essential components of the intervention were identified and included on a fidelity observation instrument to focus on the delivery of critical information from each lesson. The purpose of the treatment fidelity measure was to ensure that the intervention was implemented as intended. We observed all lessons, which were also videotaped, in the intervention classrooms and analyzed them to obtain an implementation profile for each teacher.

Metacognitive questionnaire. This measure evaluated students' metacognitive skills. We used a modified version of the metacognitive questionnaire developed by Fuchs et al. (2003). Questions from the original measure were retained if they applied to the context of our study (e.g., When I work on a math word problem, reading the words makes it hard for me to find the answer). In contrast, questions that were not specific to the context of our study were changed (e.g., "I liked doing Hot Math this year" was changed to "I liked doing percent and percent of change problems this year.") For the SBI-SM group, the questionnaire included 16 items (including 1 sample item) based on a three-point scale of "True," "Kind of True," and "Not true. Three additional questions asked students to identify the steps on the self-monitoring checklist that they knew how to do well, the solution strategies they could easily use, and the problem types they thought were the most fun to solve. For the control students, the measure focused on general questions about how the students thought about math (e.g., I am good at math; I learned a lot about math problem solving this year; I look forward to finding out if my math PSSA scores went up; when I come to a new kind of math problem, I know how to see if it is similar to a problem I have seen before) and employed the same three-point scale. Students completed this questionnaire at posttest only. For the SBI-SM, Cronbach's alpha for the 15-item test was 0.70. For the control group, Cronbach's alpha for the 10-item test was 0.52.

Treatment acceptability rating. Teachers and students completed modified versions of the Treatment Acceptability Rating Form - Revised (TARF-R) (Reimers & Wacker, 1988) at posttest only. The revised rating forms measured teacher and student acceptability of the intervention, as well as the perceived effectiveness of the intervention. The student version included nine items (e.g., I found the diagrams to be helpful in understanding and solving word problems; I would recommend using the checklists with other students my age; I am going to continue to use the diagrams to solve word problems in my classroom); whereas the teacher form consisted of 27 items (e.g., Most students liked the FOPS problem solving strategy and checklist; the FOPS problem solving strategy and checklist would be an appropriate intervention for a variety of students; the teaching scripts and materials were helpful in implementing the intervention; the diagrams are a good way to handle students' word problem solving difficulties.) The TARF-R is based on a five-point scale (i.e., strongly disagree, disagree, unsure, agree, strongly agree). Reimers and Wacker (1988) report reliability coefficients ranging from .80 to .91 for the TARF - R. On this sample, Cronbach's alpha for the teacher rating form was 0.89. For the student rating form, Cronbach's alpha was 0.76.

Professional Development

Teachers assigned to the SBI-SM condition attended a 1-day session that described the goals of the study and how to mediate instruction and facilitate discussions and group activities. In addition, ongoing technical assistance was available to the teachers throughout the duration of the study. The professional development materials included: (a) presenting ratio and proportion problems from the TIMSS and NAEP assessments and requiring teachers to solve them followed by a discussion of how their students would approach these problem types as well as analyzing student solutions and explanations, (b) demonstrating the underlying structure of the problem types by using schematic diagrams to organize the information in the problem (i.e., ratio) to highlight the essential features (e.g., compared, referent, ratio value), (c) discussing how to introduce the procedures (i.e., problem identification and representation; problem solution) inherent to the SBI-SM approach and eliciting student discussions (e.g., Why is this a ratio or proportion problem?), and (d) having teachers read the lesson plans and focusing discussion on how to represent problems using schematic diagrams, explain common rules and procedures, self regulate strategy learning using checklists, and analyze students' solutions and explanations. Project team members met with each teacher individually as needed to address individual concerns. Intervention teachers completed daily feedback logs (e.g., What were the strengths and weaknesses of the lesson?; Was the lesson missing anything either in terms of content, information or support for the teacher and/or students?; Was more time needed for the lesson?; What changes would you suggest for next year?) that provided information about the adequacy of the lessons (a copy of the teacher log is in the attachments). Teachers in the control condition attended one half-day training session describing the goals of the study, the problem solving content, and how to improve student performance on the state assessment. In addition, professional development focused on implementing the standard curriculum faithfully.

Procedure

Both conditions. Students in both conditions received instruction on ratio and proportion and were introduced to the same topics (i.e., ratios, rates, solving proportions, scale drawings, fractions, decimals, and percents) during the regularly scheduled mathematics instructional period for 40 minutes daily, five days per week, for a total of 200 minutes weekly across 10 school days. Both conditions received the same amount of instruction delivered by their classroom teachers in their intact math classes. Lessons in both intervention and control classrooms were structured as follows: (a) students working individually to complete a review problem followed by the teacher reviewing it in a whole class format, (b) the teacher introducing the key concepts/skills using a series of examples, and then (c) assigning homework. Further, students in both conditions were allowed to use calculators.

SBI-SM. For the SBI-SM condition, the researcher-designed unit replaced the students' regular instruction on ratios and proportions. Lessons were scripted to provide a detailed teaching procedure (i.e., questions to ask, examples to present) for the purpose of ensuring consistency in implementing the critical content. However, rather than read the scripts verbatim, teachers were encouraged to be familiar with them and use their own explanations and elaborations to implement SBI-SM.

To solve ratio and proportion problems, students were taught to identify the problem schema (*ratio or proportion*) and represent the features of the problem situation using schematic diagrams (see Figure 2 for sample ratio diagram). Students first learned to interpret and elaborate on the main features of the problem situation. Next, they mapped the details of the problem onto the schema diagram, with representation based on schema elaboration knowledge. Finally, they solved ratio and proportion problems by applying an appropriate solution strategy (e.g., unit rate, equivalent fraction, or cross multiplication). A four-step strategy (FOPS; F – Find the problem type, O – Organize the information in the problem using the diagram, P – Plan to solve the problem, S – Solve the problem) was developed to help anchor students' learning. FOPS served as a self-monitoring checklist to help regulate student learning. For example, using Step 1 of the strategy, the teacher thinks aloud to identify the problem type by examining information (e.g., compared, referent, and ratio value) in the problem (ratio) to recognize it as a ratio or proportion problem. For Step 2, the teacher demonstrates how to organize the information using the schematic diagram (e.g., ratio). In Step 3 of the plan, the teacher directly teaches a variety of solution methods (cross multiplication, equivalent fractions, unit rate strategies) to solve the word problems. Finally, Step 4 has the students solve the problem using the solution strategy identified in Step 3, justify the derived solutions using the schema features as anchors for explanations and elaborations, and check the accuracy of not only the computation but also the representation.

The instructional approach encouraged student "think-alouds" to foster the development of metacognitive skills. For example, self-monitoring of the strategy entailed students reflecting on their understanding of the problem (e.g., "Why is this a *proportion* problem?"), focusing on both similarities and differences between the new problem and previously learned problems; knowing *what* schematic diagram can help organize the information in the problem, *what* solution strategy can be used for solving the given problem and *why* it is appropriate. Finally, students reflected on their understanding of the solution process by asking questions (e.g., “Does it make sense? How can I verify the solution?”).

Instructional support in SBI-SM was gradually faded within and across lessons. That is, each lesson included an instructional paradigm of teacher-mediated instruction followed by guided learning and independent practice in using schematic diagrams and SM checklists as students learned to apply the learned concepts and principles. Across lessons, schematic diagrams were faded so that they were short-hand styles of the original diagrams; yet, they maintained the underlying problem structure to ensure that students continued to effectively represent the information in the problem.

Control. Students in the control group received instruction from their teachers who used procedures outlined in the district-adopted mathematics textbook (Bailey et al., 2004). *(More to be written in this section.)*

Data Analysis

For the mathematics problem solving measure, we conducted a 2 group (SBI-SM, control) x ability level (high, average, low) analysis of variance (ANOVA) to examine initial group comparability. Treatment group and ability level served as the fixed factors. To assess the acquisition and maintenance effects of the problem solving skill, we carried out separate two-factor analysis of covariance (ANCOVA) on the problem solving posttest and delayed posttest scores. Treatment group and ability level served as the between-subjects fixed factors and pretest scores served as the covariate. ANCOVA was selected to reduce the probability of a Type II error and to increase power by reducing the error variance (StatSoft, 1998).

To estimate the practical significance of effects, we computed pretest effect sizes (*ESs)* as the difference between the means divided by the pooled standard deviation (Hedges & Olkin, 1985). However, posttest effect sizes (Cohen's *d*) were calculated by dividing the difference between the regressed adjusted means (i.e., adjusted for the pretest covariate) by the square root of the mean square error (Glass, McGaw, & Smith, 1981). Further, we provide descriptive statistics for the treatment fidelity, metacognitive skills, and treatment acceptability ratings data.

Results

Table 3 presents the mean scores, adjusted mean scores, and standard deviations on problem solving tests by condition and ability level status.

Pretreatment Comparability

The ANOVA applied to the pretest scores indicated no statistically significant main effect for group, F (1, 142) = 1.10, p = .30, indicating group equivalency before the beginning of the study. However, there was a statistically significant main effect for ability level, F (2, 142) = 27.692, p < .000. Post-hoc analyses using the Bonferroni post hoc criterion for significance indicated that the mean problem solving scores for the ability levels were significantly different (High > Average > Low). No significant interaction between group and ability level was found, F (1, 142) = 0.26, p = .77.

Differential Word Problem Solving Learning as a Function of Treatment

Results of the ANCOVA applied to the posttest scores demonstrated statistically significant main effects for group, F (1, 141) = 6.30, p = .01, and ability level, F (2, 141) = 16.53, p < .000 (see Table 4). The pretest was found to be a significant covariate, F (1, 142) = 32.16, p < .000. The adjusted mean scores indicated that the SBI-SM group significantly outperformed the control group. A low medium effect size of .45 was found for SBI-SM when compared with control. Post-hoc analyses using the Bonferroni post hoc criterion for significance indicated that the mean problem solving scores for the ability levels were significantly different

(High > Average > Low). No significant interaction between group and ability level was found, F (1, 141) = 2.01.12, p = 0.14.

In addition, results from the delayed posttest administered four months following the completion of the intervention indicated statistically significant effects for group, F (1, 135) = 8.99, p = .00, and ability level, F (2, 135) = 24.16, p < .000 (see Table 5). The pretest was found to be a significant covariate, F (1, 135) = 34.06, p < .000. The adjusted mean scores indicated that the SBI-SM group significantly outperformed the control group. A medium effect size of .56 was found for SBI-SM when compared with control. Post-hoc analyses using the Bonferroni post hoc criterion for significance indicated that the mean problem solving scores for the ability levels were significantly different (High > Average > Low). No significant interaction between group and ability level was found, F (1, 135) = 2.04, p = 0.13.

Treatment Fidelity

Mean treatment fidelity across lessons for the four teachers was 79.78% (*range* = 60% to 99%), indicating moderate levels of implementation. (*Additional sections to be written on students' metacognitive skills and treatment acceptability ratings*.)

Discussion

A goal of the present study was to investigate the benefits of an instructional intervention specifically geared toward low achieving students to aid in word problem solving. The SBI-SM approach is relatively unique in its synthesis of best practices from the at-times conflicting special education and mathematics education literatures. On the one hand, SBI-SM does use direct instruction; however, it also emphasizes and highlights the underlying structure of problems as an aid to problem solving and the development of mathematical understanding.

The present results indicated that the SBI-SM intervention led to significant gains in problem-solving skills for students of varying ability levels, suggesting that it represents one promising approach to teaching ratio and proportion word problem solving skills. Our use of direct instruction, but modified to move students beyond rote memorization of conventional problem solving procedures to developing deep understanding of the mathematical problem structure and fostering flexible solution strategies, helped students in the SBI-SM group improve their problem solving performance. In addition, our results indicate that the benefits of SBI-SM persisted four months after the intervention.

These results build upon the success of our previous studies on SBI (e.g., Jitendra et al., 1998; Jitendra et al., 2007; Xin et al., 2005) in several important ways. First, although the present study found somewhat smaller effect sizes in favor of the treatment group on the delayed post-test than have been found in prior work, the delay used here (of four months) was longer than in prior SBI studies. Second, the effect size found on the delayed posttest was larger that what was found a posttest, which is consistent with prior findings. And third, the focus here on ratio and proportion problems extends into middle school our prior work on word problem solving in other mathematical domains from the elementary curriculum.

The results of the present study are particularly noteworthy given the emergence of three issues that worked against the success of SBI-SM. First, our observations of classroom instruction indicated that one of the control teachers in the high ability classroom (a very experienced teacher who only taught control classes) chose to deviate from the textbook presentation in ways that aligned with the instructional intervention. In particular, he chose to present students with *multiple* solution strategies for solving ratio and proportion problems, rather than focus on the single strategy provided in the text (e.g., cross multiplication). Second, one of the intervention teachers in the low ability group experienced classroom management difficulties that appeared to have a negative impact on student achievement. And third, implementation fidelity was, on the whole, lower than we had hoped. Despite these issues, SBI-SM still proved to be quite successful, with a low medium effect size. However, in future research with teachers implementing this intervention (particularly in larger studies, over an extended period of time), attention may need to be given to preemptively addressing issues such as these.

A related issue that merits consideration as our investigation of SBI-SM continues is whether the short professional development provided to treatment teachers (one day, followed by on-going and optional technical assistance) is sufficient. Additional professional development could be used to improve treatment fidelity. More in-depth professional development might also help allay some teachers' initial discomfort with the scripted nature of the SBI-SM curriculum. Also, increasing professional development would allow teachers and researchers to think more carefully about how SBI-SM may need to be differentiated to meet the needs of students with differing ability levels.

It is worth noting that the present intervention was time-based (with a fixed 10-day length of instruction) rather than criterion-based (with length of instruction focusing more on students' mastery of content). A time-based study may not serve the needs of some low achieving students, who may need more time and perhaps even more explicit instruction to learn the content. As such, our future research will investigate whether the addition of an *ad hoc* pull-out tutoring component for some low achieving students to help them catch up with their normally achieving peers.

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Table 1

Student Demographic Characteristics by Condition for Each Ability Level Status

	Condition							
	SBI-SM (n = 70)				Control (n = 78)			
Ability level status and characteristics	*M*	*SD*	*n*	(%)	*M*	*SD*	*n*	(%)
Academic (High)								
Age (in months)	150.84	5.04			153.48	4.32		
Gender:								
Male			7	32%			8	35%
Ethnicity								
American Indian			0	0%			0	0%
Asian			0	0%			2	9%
African American			4	18%			4	17%
Hispanic			4	18%			3	13%
White			14	64%			14	61%
Free/Subsidized Lunch			7	32%			7	30%
Special Education Status			0	0%			0	0%
Mathematics IEP			0	0%			0	0%
Reading IEP			0	0%			0	0%
ELL			1	5%			0	0%
Applied (Average)								
Age (in months)	153.12	4.32			153.48	6.72		
Gender:								
Male			19	53%			20	51%
Ethnicity								
American Indian			I	3%			0	0%
Asian			0	0%			0	0%
African American			5	14%			12	31%
Hispanic			9	25%			10	26%

White			21	58%			17	44%
Free/Subsidized Lunch			15	53%			20	51%
Special Education Status			7	19%			2	5%
Mathematics IEP			6	17%			2	5%
Reading IEP			6	17%			2	5%
ELL			1	3%			1	3%
Essential (Low)								
Age (in months)	155.88	7.80			153.12	6.48		
Gender:								
Male			6	50%			9	56%
Ethnicity								
American Indian			0	0%			0	0%
Asian			0	0%			1	6%
African American			3	25%			4	25%
Hispanic			4	33%			2	13%
White			5	42%			9	56%
Free/Subsidized Lunch			7	58%			7	44%
Special Education Status			3	25%			3	20%
Mathematics IEP			3	25%			3	20%
Reading IEP			3	25%			2	13%
ELL			2	17%			0	0%
Across all classes								
Age (in months)	152.76	5.46			153.36	5.96		
Gender:								
Male			32	46%			37	47%
Ethnicity								
American Indian			1	1%			0	0%
Asian			0	0%			3	4%
African American			12	17%			20	26%
Hispanic			17	24%			15	19%
White			40	57%			40	51%

Free/Subsidized Lunch	29	41%	33	42%
Special Education Status	10	14%	5	6%
Mathematics IEP	10	14%	5	6%
Reading IEP	10	14%	4	5%
ELL	4	6%	1	1%

Note: IEP = Individulized Education Plan; ELL = English language learner.

Table 2

Scope and Sequence of Ratio and Proportion Unit

Lessons	Content and Unit Objectives	Vocabulary
1	Ratios *Define ratio as a multiplicative relationship.* *Identify the base quantity for comparison of quantities involving part-to-part and part-to-whole*	Base quantity; back term; compared quantity; front term; part-to-part ratio; part-to-whole ratio; ratio; value of the ratio/ratio value
2	Equivalent ratios; Simplifying ratios *Use visual diagrams to understand the meaning of equivalent ratios* *Identify integer ratios in their lowest or simplest form* *Determine if ratios are in simplest form by using division of common factors*	Equivalent ratios; simplifying ratios
3 & 4	Ratio word problem solving • Apply ratio concepts to solve word problems • Represent information in the problems using a ratio schematic diagram(s) • Plan to solve the problem using a cross multiplication and equivalent fraction strategies	Cross multiplication using ratios strategy; equivalent fraction strategy; multiplicative relationship

5	Rates and Quiz • Define rate as a comparison of two quantities with different units • Understand and learn how to calculate unit rates • Learn to solve problems in which two rate are compared	Rate; unit rate; equivalent rate
6 & 7	Proportion word problem solving • Apply ratio/rate concepts to solve proportion problems • Represent information in the problems using a proportion schematic diagram(s) • Plan to solve the problem using a unit rate strategy as well as cross multiplication and/equivalent fraction strategies	If-Then Statement; proportion, unit rate strategy
8 & 9	Scale drawing word problem solving • Identify a proportional relationship in scale drawings and calculate dimensions in a scale drawing using a scale factor • Represent information in the scale drawing problems using a proportion diagram	Scale drawing/scale model; scale factor

	• Plan to solve the problem	
10	Fractions and percents • Identify percent is a special type of ratio • Understand that fractions and percents are two ways to compare parts to a whole • Understand the relationship between fractions and percents • Display the ability to convert between fractions and percents	Percent, fraction

Table 3

Mathematics Problem-Solving Performance by Condition and Students' Ability Level Status

	Condition			
	Treatment (n = 70)		Control (n = 78)	
Variable	*M*	*SD*	M	*SD*
Academic (High)				
Pretest	11.45	3.41	12.61	3.17
Posttest	15.32	2.70	14.48	2.54
Adjusted posttest	14.44		13.14	
Delayed posttest	16.41	1.50	15.38	2.22
Adjusted delayed posttest	15.51		13.00	
Applied (Average)				
Pretest	8.19	3.18	8.77	3.30
Posttest	13.03	3.00	10.95	3.11
Adjusted posttest	13.45		11.14	
Delayed posttest	12.71	3.12	11.75	3.38
Adjusted delayed posttest	13.22		11.11	
Essential (Low)				
Pretest	6.75	3.11	6.79	2.73
Posttest	8.58	3.20	8.62	2.94
Adjusted posttest	9.58		9.59	
Delayed posttest	8.10	3.63	8.00	3.52
Adjusted delayed posttest	8.88		8.30	
Across				
Pretest	8.97	3.65	9.50	3.76
Posttest	12.99	3.68	11.51	3.58
Adjusted posttest	12.49		11.29	
Delayed posttest	13.24	3.88	11.35	3.89
Adjusted delayed posttest	12.54		11.04	

Table 4

Analysis of Covariance for Problem Solving at Time 1

Source	df	F	η^2	p
Treatment Group	1	6.30**	.043	.013
Ability Level	2	16.53***	.19	.001
Treatment Group X Ability Level	2	2.01	.03	.138
Covariates				
Pretest Score	1	32.164***	.19	.001
Error	141	(7.05)		

Note. R^2 = .50 for the Total Model. Values enclosed in parentheses represent mean square errors.
*p < .05, **p < .01, ***p < .001.

Table 5

Analysis of Covariance for Problem Solving at Time 2

Source	df	F	η^2	*p*
Treatment Group	1	8.99**	.06	.003
Ability Level	2	24.16***	.26	.001
Treatment Group X Ability Level	2	2.04	.03	.134
Covariates				
Pretest Score	1	34.06***	.20	.001
Error	135	(7.20)		

Note. R^2 = .57 for the Total Model. Values enclosed in parentheses represent mean square errors.
*$p < .05$, **$p < .01$, ***$p < .001$.

Figure Captions

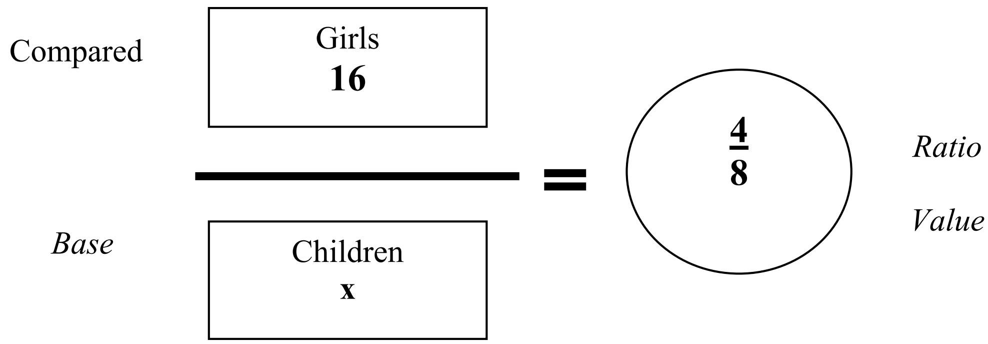
Figure 1. Sample item from PS test.

Figure 2. Teacher "think-aloud" for solving a ratio word problem

If there are 300 calories in 100 g of a certain food, how many calories are there in a 30 g portion of this food?

A. 90
B. 100
C. 900
D. 1000
E. 9000

The ratio of the number of girls to the total number of children in Ms. Robinson's class is 4:8. The number of girls in Ms. Robinson's class is 16. How many children are in the class?



Math	Explanation
	First, I figured this is a ratio problem, because it compared the number of girls **to** the number of children in Ms. Robinson's class. This is a **part-whole ratio** that tells about a **multiplicative relationship** (4:8) between the number of girls and children.
$\frac{16 \text{ girls}}{x \text{ children}} = \frac{4}{8}$	**Next**, I represented (or organized) the information in the problem using a ratio diagram and set up the math equation.
	Then, I decided to use **cross multiplication using ratios strategy** to solve for the number of children.
$\frac{16 \text{ girls}}{x \text{ children}} = \frac{4}{8}$ 4 * x = 16 * 8 4x = 112 So 112 ÷ 4 is 32 **Answer: <u>32 children are in Ms. Robinson's class.</u>**	**Finally,** I cross multiplied and got 4 * x = 16 * 8 4x = 112 x = 32 So, 32 is the number of children in Ms. Robinson's class.