PSYCHOMETRIKA—VOL. 50, NO. 2, 203–228
JUNE 1985

UNDERSTANDING CORRELATES OF CHANGE BY MODELING INDIVIDUAL DIFFERENCES IN GROWTH

DAVID R. ROGOSA AND JOHN B. WILLETT

STANFORD UNIVERSITY

The study of correlates of change is the investigation of systematic individual differences in growth. Our representation of systematic individual differences in growth is built up in two parts: (a) a model for individual growth and, (b) a model for the dependence of parameters in the individual growth models on individual characteristics. First, explicit representations of correlates of change are constructed for various models of individual growth. Second, for the special case of initial status as a correlate of change, properties of collections of growth curves provide new results on the relation between change and initial status. Third, the shortcomings of previous approaches to the assessment of correlates of change are demonstrated. In particular, correlations of residual change measures with exogenous individual characteristics are shown to be poor indicators of systematic individual differences in growth.

Key words: longitudinal analysis, measurement of change.

The assessment of correlates or predictors of change is motivated by research questions such as "What kinds of persons grow (learn) fastest?" (Cronbach & Furby, 1970, p. 77). Many of the controversies regarding the use of "change scores" arise in their use as criteria in correlational studies. This paper strives to resolve such controversies and to provide a framework for productive analyses of correlates of change. We build upon the mathematical framework for the analysis of change developed in Rogosa, Brandt and Zimowski (1982) and follow the philosophy that "Individual time paths are the proper focus for the analysis of change" (Rogosa et al., 1982, p. 722).

The basis for our understanding of correlates of change is a two-part representation of systematic individual differences in growth using a pair of statistical models: (a) a model for individual growth and (b) a model for the dependence of parameters in the individual growth models on individual characteristics. In order to address the question, What kinds of persons grow (learn) fastest?, it is necessary to specify the form of individual growth and also, to specify how individual differences in growth depend upon the individual characteristic(s) that distinguish different "kinds" of persons.

The first section of the paper describes some possible representations of systematic individual differences in growth. Following a tradition in the statistical analysis of growth, the growth of an individual is summarized by the parameter values of the individual growth curve. Part (a) of our representation specifies a growth curve for each individual, where the parameters of the individual growth curves may differ across individuals. Such statistical models for collections of individual growth curves are a common special case of regression models with random coefficients (see, for example, Fisk, 1967; Rao, 1965; Swamy, 1971, 1974). However, with the random coefficient models, the emphasis has been on the mean growth curve or on the variances across individuals of parameters of the individual regression models (indicating the amount of individual differences in growth). Part (b) of our representation makes the individual differences in growth "systematic" by specifying a statistical model for the parameter(s) of the individual growth curves as a

The research reported here was supported by a grant from The Spencer Foundation to the senior author and by the Study of Stanford and the Schools.

Requests for reprints shoulc be sent to David Rogosa, School of Education, Stanford University, Stanford CA 94305.

0033-3123/85/0600-6044 $00.75/0

function of the correlate(s) of change. In addition, we contrast individual differences in the amount of change (which is all that can be estimated when only two observations on each individual are available) with individual differences in the parameters of the growth models.

In investigations of correlates of change, two kinds of measures have been used as the correlate: either (i) initial status on the outcome variable, or (ii) an exogenous, background characteristic. For example, numerous empirical studies have investigated the correlation between change and initial status on achievement or on intelligence (Bloom, 1964; Thorndike, 1966; Werts & Hilton, 1977). In particular, Bloom (1964) provides a compendium of empirical results on the correlation between change and initial status. Typical examples of exogenous individual characteristics as correlates of change in intelligence include measures of personality and parental behavior in McCall, Appelbaum, and Hogarty (1973), SES in Rees and Palmer (1970), home background variables in Harnquist (1968), environmental measures in Bloom (1964), or measures of personality and behavior in Bayley (1968). Wohlwill (1980) provides an overview of many such studies. Also, the "process-product" paradigm in research on teaching provides numerous examples of correlates of student learning using measures of teachers' classroom behavior as the exogenous variable (Berliner, 1976; Borich, 1979; Medley, 1979; Veldman & Brophy, 1974). The relation between change and initial status and the relation between change and an exogenous variable are the topics of the second and third sections, respectively.

The special case of initial status as the correlate of change is of considerable interest because the correlation between change and initial status is the basis for a variety of substantive interpretations. The Law of Initial Values (Campbell, 1981; Lacey & Lacey, 1962; Wilder, 1965) prescribes that the correlation between change and initial status is negative. The Overlap Hypothesis (Anderson, 1939; Bloom 1964; Cronbach & Snow, 1977; Roff, 1941) specifies that the correlation between change and initial status is zero. Furthermore, conditions for regression toward the mean (Furby, 1973; Nesselroade, Stigler & Baltes, 1980), and fan spread (Cook & Campbell, 1979) may be stated in terms of the correlation between change and initial status. The results for properties of collections of growth curves in the second section express the relation between change and initial status as a function of the time of initial status, allowing an integrated interpretation of these diverse topics.

A long tradition of detailed examination of individual development and learning (and individual differences in development and learning) over multiple occasions of measurement exists in the behavioral sciences (Bayley, 1949, 1956; Lewis, 1960; Woodrow, 1940). Regrettably, the procedures for the assessment of correlates of change prescribed by the measurement of change literature abandon this tradition, using neither models for individual growth nor models for individual differences in growth. Instead, the exogenous individual characteristic is correlated with, or used to predict, a "measure of change" (e.g., a residual change score). Although elaborate procedures have been developed to counter the effects of errors of measurement (Tucker, Damarin, & Messick, 1966), the use of these measures remains problematic. Not only do these procedures lack a specification of individual growth, the measures of change are almost always obtained from observations on only two occasions. (In fact, many of these procedures become intractable when extended to multiple occasions.) The results in the third section demonstrate the shortcomings of these popular procedures for the assessment of correlates of change, especially when some form of residual change score is used.

1. What is a Correlate of Change?

The first step in our formulation of correlates of change is the statement of the individual growth curve; that is, it is necessary to define the functional form of growth before

considering individual differences in growth. First, we examine an individual growth model having a constant rate of change. For this model we complete a basic representation of systematic individual differences in growth by relating individual differences in an exogenous individual characteristic to individual differences in the rate of change. Subsequently, we consider systematic individual differences in growth for growth models (e.g., polynomial, exponential, logistic, simplex) which have non-constant rates of change.

To set notation, for an attribute ξ, $\xi_p(t)$ defines the form of the growth curve for individual p. The instantaneous rate of change in ξ is $d\xi/dt$ (the first derivative of $\xi_p(t)$ with respect to time). The amount of change in ξ for individual p over the time interval $[t, t + \tau]$ is denoted by $\Delta_p(t, t + \tau)$ which equals $\xi_p(t + \tau) - \xi_p(t)$. In empirical studies, fallible measures of ξ, which we denote by X, are obtained at discrete time points t_i (where, as in panel data, observations are obtained at occasions of measurement $t_1, t_2, \ldots, t_T$).

We denote a generic, unchanging exogeneous individual characteristic by W. For simplicity, we speak in terms of a single W, although multiple W's can be accommodated. Also, W is interpreted as measured without error. The complications induced by recognizing that W may be fallible are not important for understanding correlates of change. However, statistical estimation of many of the parameters identified in this paper requires attention be given to the measurement properties of W.

1.1 Systematic Individual Differences in Straight-Line Growth

The simplest type of individual growth curve specifies that each individual has a constant rate of change. The rate of change for the pth individual is θ_p. This constant rate of change model implies a straight-line growth curve for individual p:

$$\xi_p(t) = \xi_p(0) + \theta_p t. \tag{1}$$

Individual differences in growth exist when different individuals have different values of θ_p. Systematic individual differences in growth exist when individual differences in a growth parameter such as θ_p can be linked with one or more W's. A simple representation for straight-line growth is:

$$E(\theta \mid W) = \mu_\theta + \beta_{\theta W}(W - \mu_W). \tag{2}$$

The conditional expectation, $E(\theta \mid W)$, need not be a simple linear relation as specified in (2), even for straight-line growth. An instance of a quadratic relation between growth and a W from educational research arises in interpretation of process-product correlations with process variables such as teacher praise (see Soar, 1977, 1979, especially Figures 1 and 2). Although we mainly use simple linear relations between W and parameters of models for individual growth, this approach can easily be extended to include general forms of the conditional expectation, $E(\theta \mid W)$, for a single W or a vector of W's.

Equations (1) and (2) constitute the two parts of an explicit specification for the study of correlates or change—a model for individual growth and a model for individual differences in growth. In (2) a non-zero value of $\beta_{\theta W}$ indicates that W is a predictor of growth. Often, the correlation between θ and W over individuals, $\rho_{\theta W}$, is a convenient summary quantity. For these models, both $\beta_{\theta W}$ and $\rho_{\theta W}$ address questions such as, Do some individuals (as distinguished by their values of W) learn faster than others? Or, is W a useful predictor of growth?

1.2 Systematic Individual Differences in Exponential Growth

The constant rate of change model has a central role in this paper as a convenient growth model for obtaining exact mathematical results on individual differences in growth. Moreover, in applications straight-line growth serves as a useful approximaton to actual growth processes; for example, Hui and Berger (1983) justify fitting straight-line growth curves to epidemiological data: "an important feature of many epidemiologic

studies is that the follow-up intervals of most of the individuals studied are sufficiently short that the response curves in the intervals can be approximated by straight lines" (p. 753).

Growth models with non-constant rates of change are also important in allowing more realistic modeling of growth and more complex representations of systematic individual differences in growth. Our primary example of a growth model with a non-constant rate of change specifies that the rate of change is a linear function of current status (hence the label "linear state-dependence" model). For example, this model specifies that the rate of learning for individual p is proportional to the amount yet to be learned:

$$\frac{d\xi}{dt} = (\lambda_p - \xi)\gamma_p. \tag{3}$$

In this model, the parameter λ_p represents the ceiling or asymptote on ξ for individual p; for example, if $\xi_p(t)$ represents the level of academic achievement for individual p at time t, then $\lambda_p - \xi_p(t)$ represents the amount yet to be learned. The growth curve implied by (3) exhibits negatively accelerated growth to the asymptote:

$$\xi_p(t) = \lambda_p - (\lambda_p - \xi_p(0))e^{-\gamma_p t}. \tag{4}$$

In many applications γ_p is referred to as the "learning rate constant" as, the time taken to grow by a fraction f of the amount yet to be learned, $\lambda_p - \xi_p(t)$, is $-(1/\gamma_p)\ln(1-f)$. Psychological applications of this model are widespread, one of the most familiar being Hull's (1943) use of the curve to model "habit strength" (see also Hilgard, 1951). Hicklin (1976) also applies variants of this model to the study of mastery learning. (See also Keats, 1983; Sagiv, 1979).

The parameters λ_p and γ_p may differ over individuals. Formulations of systematic individual differences in growth are completed by models for the dependence of λ and γ on one or more W's. Various forms for this dependence can be constructed. For example with two distinct exogenous individual characteristics, W_1 and W_2, individual differences in γ may depend on W_1, whereas individual differences in λ may depend upon W_2. Alternatively, λ and γ may both depend on the same W or collection of W's.

Coleman (1968) introduced models for exogenous influences on change which are widely used in sociology. In our notation, the model in Coleman (1968, Equation 11.15) can be written:

$$\frac{d\xi}{dt} = (\mu_\lambda - \xi)\gamma + \beta_{\lambda W}(W - \mu_W)\gamma. \tag{5}$$

The model for the rate of change in (5) can be obtained by substituting the relation $\lambda_p = \mu_\lambda + \beta_{\lambda W}(W - \mu_W)$ into a restricted form of (3) in which $\gamma_p = \gamma$ for all individuals. Thus, Coleman's model may be considered a specialized formulation of systematic individual differences in exponential growth that invokes the assumptions: (a) $\gamma_p = \gamma$ and (b) $\rho_{\lambda W} = 1$.

Coleman's formulation exemplifies an alternative tradition in the investigation of influences of exogenous variables on growth (i.e., correlates and predictors of change). In the application of these models to a collection of individuals, the model (5) specifies that the parameters are identical for all individuals; an assumption described by Coleman as "the process is identical for all persons" (1968, p. 437). Individual differences in $d\xi/dt$ enter into Coleman's model only through individual differences in the exogenous variables. One empirical analysis based on Coleman's formulation is the analysis of school learning by Sørensen and Hallinan (1977) in which two exogenous individual characteristics (student effort and student ability) were incorporated into the model for exogenous influences on

change in (5). Additional examples in this tradition are Tuma and Hannan (1984, chap. 11) and Nielsen and Rosenfeld (1981). Salemi and Tauchen (1982) provide another instance of models for influences on change in which the parameters of the growth function do not vary over individuals. In their analysis of the learning of principles of economics by community college students, the exogenous individual characteristics are a variety of demographic and educational measures which are incorporated into the model to allow individual differences in learning.

1.3 Alternative Non-Constant Rate of Change Models

In general, for growth models with a non-constant rate of change, $d\xi/dt$ depends on t (time dependence) and on the current level of ξ (state dependence). General forms for non-constant rate of change models are presented by Day (1966), Guire and Kowalski (1979), Richards (1959), Sandland and McGilchrist (1979). Three further examples of non-constant rate of change models—polynomial growth curves, the logistic curve and simplex models—which have been used in many applications are briefly considered below.

Polynomial growth curves. The use of polynomial growth curves in the statistical literature has been common since Wishart (1938). An obvious extension of the straight-line growth model is to represent $\xi_p(t)$ by a polynomial of degree 2:

$$\xi_p(t) = \xi_p(0) + \eta_{1p}t + \eta_{2p}t^2, \tag{6}$$

for which the rate of change, $d\xi/dt = \eta_{1p} + 2\eta_{2p}t$, exhibits a time dependence. A convenient approach for dealing with a time-dependent rate of change is to use an average rate of change. Seigel (1975) and Hui and Berger (1983) demonstrate that the average rate of change for a polynomial growth curve of degree 2 is simply θ_p in the straight-line growth model. (Note that η_{1p} differs from θ_p except for values of t near 0.) Consequently, even when the correct growth model is a polynomial of degree 2, individual differences in θ_p are of interest. Thus, the quantities $\rho_{\theta W}$ and $\beta_{\theta W}$ can be used to represent systematic individual differences in (average) growth.

Logistic growth. A growth model having an inflection point at $\xi_p(t) = \lambda_p/2$ and upper asymptote λ_p is:

$$\frac{d\xi}{dt} = \xi\left(1 - \frac{\xi}{\lambda_p}\right)\gamma_p,$$

which implies a logistic growth curve. Individual differences in either λ_p or γ_p may depend on exogenous individual characteristics. One ambitious empirical attempt to study individual differences in the parameters of a logistic learning curve is Woodrow (1940); Thurstone and Ackerson (1929) is an early application of the logistic growth function.

Simplex models. A different type of non-constant rate of change model arises in the generation of the simplex correlation structure. In many analyses of longitudinal data a simplex correlation structure serves as the basic model for statistical analysis (for example, Anderson, 1960; Guttman & Guttman, 1965; Humphreys, 1960; Jöreskog, 1970; Werts, Linn & Jöreskog, 1977). Following Jöreskog (1970), a discrete-time autoregressive model which gives rise to the simplex correlation structure has the form:

$$\xi_p(t_{i+1}) = B_i\,\xi_p(t_i) + \delta_{ip} \tag{7}$$

where the discrete times of observation are indicated by $t_1, t_2, \ldots, t_T$ and the δ_{ip} are independent disturbances for which $E(\delta_{ip}) = 0$. Growth represented by (7) can be seen to have a non-constant rate of change (even for $B_i = B$) from the expression for the amount of change in the time interval $[t_i, t_{i+1}]$:

$$\Delta_p(t_i, t_{i+1}) = (B_i - 1)\xi_p(t_i) + \delta_{ip}.$$

As only the δ_{ip}, and not the B_i, may differ across individuals, the simplex model does not permit representations of systematic individual differences in growth.

1.4 Individual Differences in Amount of Change

Through the various growth models introduced in this section individual differences in growth are represented by individual differences in the parameters of a particular growth model. Systematic individual differences in growth are said to exist when one or more parameters of the growth model depend on exogenous individual characteristics. Systematic individual differences in growth cannot exist unless at least one of the parameters of the individual growth model differs among individuals.

An alternative, and more common, approach to studying correlates of change focuses simply on the total amount of change in a specific time interval. Instead of individual differences in the parameters of a growth model, individual differences in the amount of change, $\Delta_p(t, t + \tau)$, are central. Only for the constant rate of change model, for which $\Delta_p(t, t + \tau) = \tau\theta_p$ and thus $\rho_{\Delta W} = \rho_{\theta W}$, are representations of individual differences in terms of the amount of change interchangeable with representations in terms of the parameters of the individual growth model. With non-constant rate of change models the relation between the parameters of the growth model and the amount of change may be complex. Most important, this relation may depend strongly on t or τ.

To illustrate the consequences and potential pitfalls of studying individual differences in the amount of change, consider the exponential growth model in (4). For this growth model the amount of change in the interval $[t, t + \tau]$ is:

$$\Delta_p(t, t + \tau) = e^{-\gamma_p t}(\lambda_p - \xi_p(0))(1 - e^{-\gamma_p \tau}). \tag{8}$$

Clearly, the amount of change depends on both t and τ. Most important, the dependence is such that the rank order of individual change may be altered considerably for different choices of t: e.g., $\Delta(t_1, t_1 + \tau)$ versus $\Delta(t_2, t_2 + \tau)$. The volatility of individual differences in the amount of change may be seen more clearly by examining $\ln(\Delta_p)$:

$$\ln[\Delta_p(t, t + \tau)] = -\gamma_p t + \ln[\lambda_p - \xi_p(0)] + \ln[1 - e^{-\gamma_p \tau}].$$

Only when no individual differences exist in γ_p ($\gamma_p = \gamma$ for all p) will the rank order of the $\Delta_p(t, t + \tau)$ be maintained for all t.

Difficulties with the use of the amount of change may be especially severe for interpreting associations between Δ and exogenous individual characteristics. With appreciable individual differences in γ_p the rank order of the $\Delta_p(t, t + \tau)$ may completely reverse over time; that is, an initial rank-ordering on $\Delta_p(t_1, t_1 + \tau)$ may be *opposite* to the rank-ordering on $\Delta_p(t_2, t_2 + \tau)$. Thus, it is possible $\rho_{\Delta W}$ may indicate systematic individual differences in the opposite direction or may be near zero even in the presence of strong systematic individual differences in the parameters of the growth model. For example, strong relationships may exist between λ and W_1 and between γ and W_2 yet, $\Delta(t, t + \tau)$ may have no relation with the exogenous individual characteristics W_1 and W_2. In particular, for exponential growth a collection of (λ_p, γ_p) values exhibiting appreciable individual differences may be stipulated such that no individual differences exist in the amount of change—the $\Delta_p(t, t + \tau)$ are *identical* for all individuals.

The example above illustrates the danger of characterizing the growth of individuals by the amount of change over a specific time interval; instead, parameters of the individual growth curves define the individual differences in growth. This message is particularly important for longitudinal designs with only two observations on each individual. Two-wave designs permit at best the study of individual differences in Δ or, equivalently, in some sort of average rate of change. Consequently, designs with only two observations are usually inadequate for the assessment of systematic individual differences in growth.

2. Using Initial Status as a Correlate of Change

A large amount of the literature on the analysis of change has focused on the correlation between change and initial status. For example, the correlation between change and initial status figures prominently in the Overlap Hypothesis, the Law of Initial Values, and the Regression Effect. Regrettably, this literature on the relation between change and initial status is riddled with confusion and misinformation. The major shortcomings of the previous investigations are (a) the lack of models for individual growth, and (b) the consideration of only two waves of data (or similarly, the breaking up of data obtained at multiple time points into many two-wave pieces). We present important new information on initial status as a correlate of change obtained from mathematical analyses of the statistical properties of collections of growth curves.

Denoting the time of initial status by t_I and true initial status by $\xi(t_I)$, the central quantities for initial status as a correlate of change are $\rho_{\xi(t_I)\Delta}$, the correlation between change and initial status, and $\beta_{\Delta\xi(t_I)}$, the regression coefficient of change on initial status. (The amount of change in a time interval beginning at t_I and extending τ time units, $\Delta(t_I, t_I + \tau)$, is abbreviated to Δ.) The interpretation of the quantities $\rho_{\xi(t_I)\Delta}$ and $\beta_{\Delta\xi(t_I)}$ presumes a simple linear relation between change and initial status. For straight-line growth, this relation can be obtained by substituting $\xi(t_I)$ for W in (2). Blomqvist (1977, Equation 3.1) models the relation between change and initial status using such a simple linear relation (along with straight-line growth) in developing a maximum likelihood estimate for $\beta_{\Delta\xi(t_I)}$. Alternatively, Garside (1956, Equation 34) stated the conditional expectation, $E(\Delta \mid \xi(t_I))$, as a polynomial in $\xi(t_I)$. A serious drawback of curvilinear models for the conditional expectation, $E(\Delta \mid \xi(t_I))$, is that the form of the conditional expectation will differ, even for the same collection of growth curves, for different $t = t_I$.

First, this section presents the mathematical properties of initial status as a correlate of change for straight-line growth, the simplex model, and exponential growth. Using these results, selected empirical studies of the relationship between change and initial status are re-examined. Finally, facts and fallacies about the Regression Effect (a.k.a. Regression Toward the Mean) are explained in terms of the correlation between change and initial status.

2.1 Initial Status and Straight-Line Growth

Results are presented which describe the properties of $\rho_{\xi(t_I)\Delta}$ and $\beta_{\Delta\xi(t_I)}$ for collections of straight-line growth curves. Straight-line growth has been implicit in previous technical work which only considers measurements at two points in time. Most often, the correlation $\rho_{\xi(t_I)\Delta}$ has been the focus of empirical studies and methodological investigations. For straight-line growth, $\rho_{\xi(t)\Delta} = \rho_{\xi(t)\theta}$, and thus $\rho_{\xi(t_I)\theta}$ is used to represent the correlation between change and initial status.

Time of initial status and $\rho_{\xi(t)\theta}$. The functional dependence of $\rho_{\xi(t)\theta}$ on t reveals the consequences for $\rho_{\xi(t_I)\theta}$ of different choices of time of initial status. The crucial dependence of $\rho_{\xi(t_I)\theta}$ on the t chosen to be t_I is an important, and previously unrecognized, property of the correlation between change and initial status. Equation (9) demonstrates this dependence:

$$\rho_{\xi(t)\theta} = \frac{(t - t^\circ)}{(\kappa^2 + (t - t^\circ)^2)^{1/2}}. \tag{9}$$

Equation (9) introduces two important quantities: the "centering point" t° and the "scaling constant" $\kappa = (\sigma_{\xi(t^\circ)}/\sigma_\theta)$. Both t° and κ are properties of the particular collection of straight-line growth curves. The time t°, the time at which $\rho_{\xi(t)\theta} = 0$, serves as a centering

point for the time scale and can be defined by the relation:

$$t - t^\circ = \frac{\sigma_{\xi(t)\theta}}{\sigma_\theta^2}.$$

Thus t° equals $-\sigma_{\xi(0)\theta}/\sigma_\theta^2$ (assuming that $\sigma_\theta^2 > 0$; that is, individual differences in growth exist). One of the properties of t° is that the variance of $\xi(t)$ is minimized at $t = t^\circ$. And, from the relation

$$\frac{\sigma_{\xi(t)}^2}{\sigma_{\xi(t^\circ)}^2} = 1 + \left(\frac{t - t^\circ}{\kappa}\right)^2 \tag{10}$$

it can be seen that the "scaling constant" κ determines the rate at which $\sigma_{\xi(t)}^2$ increases from its minimum, $\sigma_{\xi(t^\circ)}^2$, as $|t - t^\circ|$ increases.

Figure 1 displays the functional form of $\rho_{\xi(t)\theta}$ given in (9). The correlation $\rho_{\xi(t)\theta}$ is a monotonically increasing function in t with asymptotes of ± 1.0. Only at $t = t^\circ$ does $\rho_{\xi(t)\theta}$ equal zero. Also, $\rho_{\xi(t)\theta} = \pm 1/(2)^{1/2}$ at $t = t^\circ \pm \kappa$. The function $\rho_{\xi(t)\theta}$ has a maximum gradient of $1/\kappa$ at $t = t^\circ$; thus the choice of time of initial status is most consequential for t_I in the neighborhood of t°. And t° appears to be reasonably near the time of initial status in much empirical research, judging from the findings of small, negative correlations between observed change and observed initial status.[1]

Overlap Hypothesis. The Overlap Hypothesis is often stated as specifying that the correlation between change and initial status is zero (see, for example, Bloom, 1964; Roff, 1941). Equation (9) demonstrates that for collections of straight-line growth curves the Overlap Hypothesis (stated in terms of true scores) can only be satisfied when t° is the time of initial status. A collection of straight-line growth curves is unlikely to satisfy the Overlap Hypothesis for an arbitrary choice of t_I.

Law of Initial Values. For any $t_I < t^\circ$, $\rho_{\xi(t_I)\theta}$ is negative. That $\rho_{\xi(t_I)\theta} < 0$ is a statement, in terms of true scores, of the Law of Initial Values (LIV). An alternative expression of LIV by Tucker, Damarin and Messick (1966) uses the relation $\beta_{\xi(t+\tau)\xi(t)} - 1 = \beta_{\Delta\xi(t)}$ to re-express LIV as $\beta_{\xi(t_I+\tau)\xi(t_I)} < 1$. As (9) and Figure 1 reveal, if t_I exceeds t° then LIV is contradicted. Thus, with straight-line growth, LIV cannot hold for all choices of t_I; in particular, a collection of straight-line growth curves which satisfy LIV at one t_I will no longer satisfy LIV if initial status is defined at a sufficiently later time. Despite these difficulties with LIV as a law-like statement, Wilder (1965) is able to point to over 400 empirical studies serving to corroborate LIV. Yet, in almost all empirical studies devoted to LIV (Lacey, 1956; Wilder, 1965), no distinction is made between true and observed scores; such confusion is regrettable because, as has been repeatedly documented in the literature, errors of measurement may produce serious negative bias in the correlation between observed change and observed initial status. Recently, Tucker (1979) and Messick (1981) have emphasized the fundamental importance of true score formulations for LIV.

Fan-spread. The term fan-spread is often used to describe $\sigma_{\xi(t)}^2$ increasing with time (Cook & Campbell, 1979, pp. 184–5; Walberg & Tsai, 1983). Clearly from (10), $\sigma_{\xi(t)}^2$ increases at later times if $t > t^\circ$ (i.e., $\rho_{\xi(t)\theta} \geq 0$). In general, for the time interval $[t, t + \tau]$, $\sigma_{\xi(t+\tau)}^2 \geq \sigma_{\xi(t)}^2$ if and only if $t \geq t^\circ - (\tau/2)$, which implies $\rho_{\xi(t)\theta} \geq -[1 + (2\kappa/\tau)^2]^{-1/2}$. The equality $\sigma_{\xi(t)}^2 = \sigma_{\xi(t+\tau)}^2$ is satisfied if and only if $t = t^\circ - (\tau/2)$; a condition of constant variance over two or more time points is often termed "dynamic equilibrium" (Kessler & Greenberg, 1981, chap. 2; Lord, 1963, p. 33). In the literature on the analysis of quasiex-

[1] The correlation of observed change and observed initial status, $\rho_{X_i(X_{i+k}-X_i)}$ will not equal zero at $t_i = t^\circ$. With just two observations on each individual, the negative bias of $r_{X_1(X_2-X_1)}$ for estimating $\rho_{\xi(t_1)\Delta}$ indicates that the correlation between observed change and observed initial status equals zero at a time greater than t°.

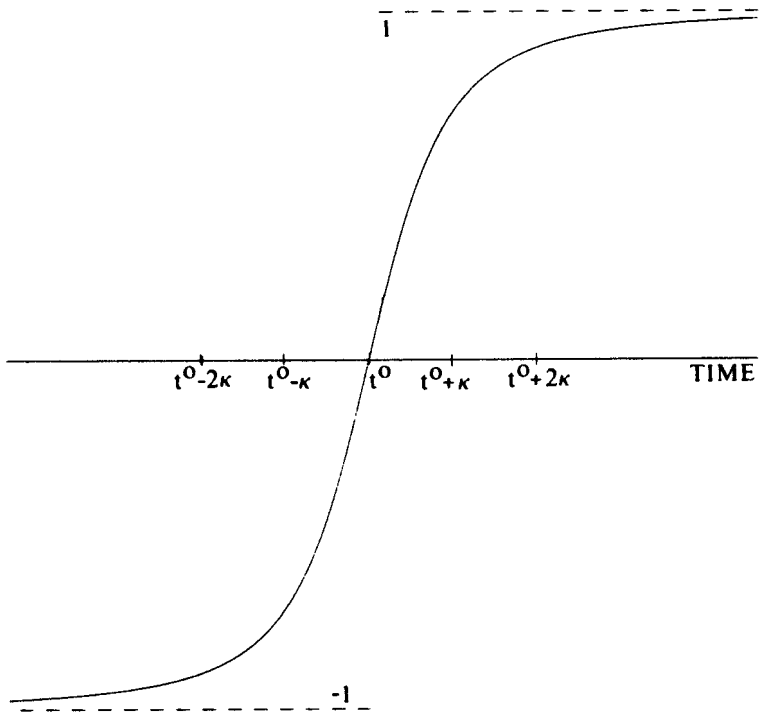

FIGURE 1
Plot of $\rho_{\xi(t)\theta}$ against time for the straight-line growth model. The asymptotes of $+1$ and -1 are indicated at the dashed lines.

periments, the term "degenerate fan spread" (Bryk & Weisberg, 1977) is used to describe a collection of straight-line growth curves (for $t > t^\circ$) having $\sigma^2_{\xi(t^\circ)} = 0$.

Regression of change on initial status. The regression coefficient $\beta_{\Delta\xi(t_I)}$ also depends crucially on the t chosen to be t_I, as demonstrated by the expression for $\beta_{\Delta\xi(t)}$ in (11):

$$\beta_{\Delta\xi(t)} = \frac{\tau(t - t^\circ)}{\kappa^2 + (t - t^\circ)^2}, \tag{11}$$

where $\Delta = \Delta(t, t + \tau)$. Figure 2 displays $\beta_{\Delta\xi(t)}$ as a function of t. The function has a maximum value of $\tau/2\kappa$ at $t = t^\circ + \kappa$ and a minimum value of $-\tau/2\kappa$ at $t = t^\circ - \kappa$. Both asymptotes of the function are zero, and $\beta_{\Delta\xi(t^\circ)} = 0$. The gradient of the function has a maximum value of τ/κ^2 at $t = t^\circ$. The properties shown by (11) have further implications by virtue of the relation $\beta_{\xi(t+\tau)\xi(t)} = \beta_{\Delta\xi(t)} + 1$.

Illustrative example. Figure 3 displays a collection of 15 straight-line growth curves whose features are used to provide further illustration of the properties of the relation between change and initial status. This collection of growth curves has centering point $t^\circ = 3.0$ and scaling constant $\kappa = 3.0$. This collection of growth curves generates a correlation structure among the $\xi(t_i)$. Table 1 presents the between-wave correlation matrix having elements $\rho_{\xi(t_i)\xi(t_{i+k})}$ for discrete times t_i, t_{i+k} which are integers between 1 and 5. The correlation matrix in Table 1 shows that this collection of growth curves yields a correlation structure (for true scores) that is consistent with correlations indicated by much longitudinal research. (The use of a small number of growth curves in Figure 3 is purely for display clarity; the correlation matrix in Table 1 may also be thought of as generated by a large collection of growth curves having the same values of t° and κ as the 15 growth curves shown.)

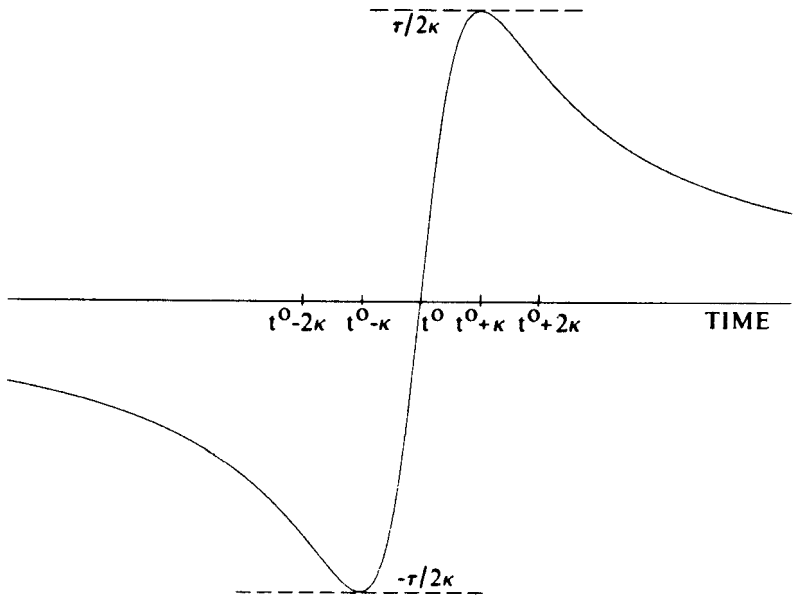

FIGURE 2
Plot of $\beta_{\Delta\xi(t)}$ against time for the straight-line growth model. Values of the maximum and minimum are indicated at the dashed lines.

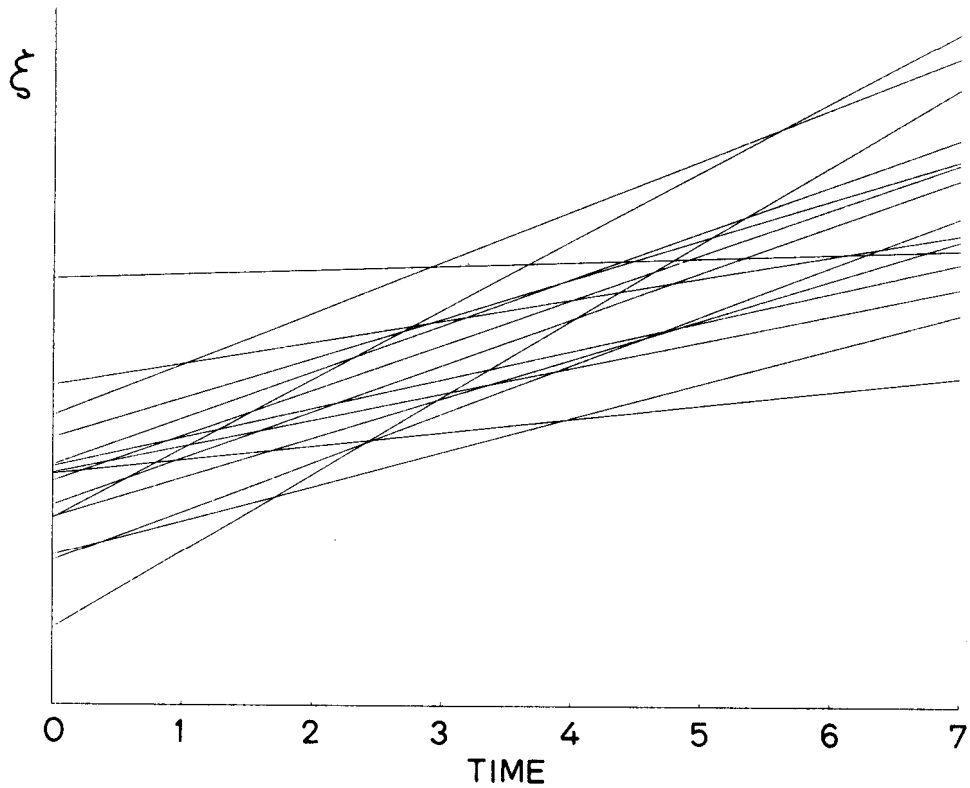

FIGURE 3
A collection of 15 straight-line growth curves in ξ having $t^\circ = 3.0$ and $\kappa = 3.0$.

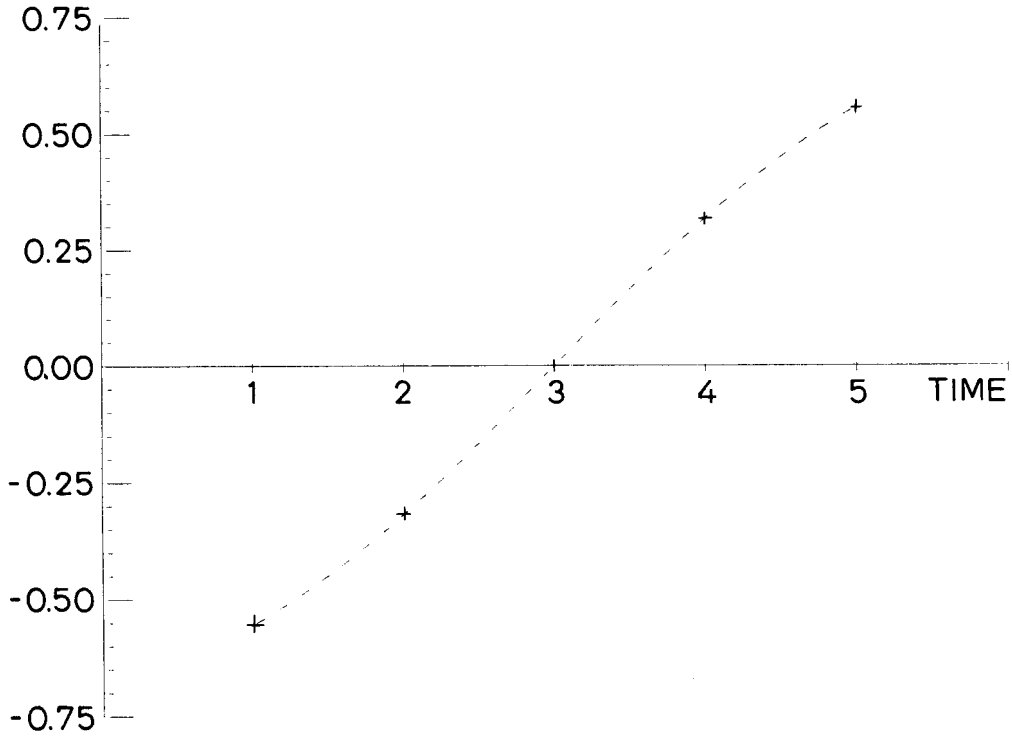

FIGURE 4
Values of the correlation between change and initial status corresponding to the collection of growth curves in Figure 3.

Figure 4 displays values of the correlation between change and initial status for initial status between times 1 and 5. In Figure 4 a vertical cross (+) indicates those values of $\rho_{\xi(t_I)\Delta}$ for t_I having the integer values used in the correlation matrix in Table 1. Figure 4 demonstrates that depending upon the choice of the time of initial status the growth curves of Figure 3 yield markedly different values of the correlation between change and initial status.

2.2 *Initial Status and the Simplex Model*

The discrete-time, autoregressive model in (7) yields an especially simple form for the regression of change on initial status:

$$\beta_{\Delta(t_i,\, t_{i+k})\xi(t_i)} = \left(\prod_{q=0}^{k-1} B_{i+q}\right) - 1, \tag{12}$$

For $k = 1$, (12) simplifies to

$$\beta_{\Delta(t_i,\, t_{i+1})\xi(t_i)} = B_i - 1.$$

Thus, for a specific t_i chosen as the time of initial status, t_I, the sign of both $\beta_{\Delta\xi(t_I)}$ and $\rho_{\xi(t_I)\Delta}$ (where $\Delta = \Delta(t_I, t_{I+k})$) is determined by whether the product of the coefficients B_{i+q} intervening between t_I and t_{I+k} is greater or less than one.

To illustrate the implications of (12), consider that for a model in which the B_i are all greater than one, $\rho_{\xi(t_i)\Delta}$ is always positive; whereas, in a model for which the B_i are all less than one, $\rho_{\xi(t_i)\Delta}$ is always negative (satisfying LIV). Also, if all the B_i are one (or, more generally, their product equals one), $\rho_{\xi(t_i)\Delta}$ is zero for all t_i, and thus the simplex model

TABLE 1

Correlation Matrix of the $\xi(t_i)$ Corresponding to the Growth Curves in Figure 3

	$\xi(t_1)$	$\xi(t_2)$	$\xi(t_3)$	$\xi(t_4)$	$\xi(t_5)$
$\xi(t_1)$	1.0	.96	.83	.61	.38
$\xi(t_2)$		1.0	.95	.80	.61
$\xi(t_3)$			1.0	.95	.83
$\xi(t_4)$				1.0	.96
$\xi(t_5)$					1.0

with $B_i = B = 1$ satisfies the Overlap Hypothesis for any t_I. This simplex model (see especially Humphreys, 1960) is equivalent to the original formulation of the Overlap Hypothesis in Anderson (1939) which specifies growth to be by the accumulation of independent increments.

The correlation between change and initial status for $k = 1$ is

$$\rho_{\Delta(t_i,\, t_{i+1})\xi(t_i)} = \frac{B_i - 1}{\left((B_i - 1)^2 + \dfrac{\sigma^2_{\delta_i}}{\sigma^2_{\xi(t_i)}}\right)^{1/2}}. \tag{13}$$

In many expositions of the simplex model, the restrictions that $\sigma^2_{\delta_i} = (1 - B_i^2)$ and $\sigma^2_{\xi(t_1)} = 1$ are invoked (Anderson, 1959, p. 210; Jöreskog, 1970, p. 141); the consequence of these restrictions is that $\rho_{\xi(t_i)\Delta}$ is negative for all t_i, by virtue of all the B_i being constrained to be less than one. Invoking the additional condition that $B_i = B$ for all i yields perhaps the only collection of (easily specified) growth curves for which $\rho_{\xi(t_I)\Delta}$ is the same for all choices of t_I.

2.3 *Initial Status and Exponential Growth*

A restricted form of (3)

$$\frac{d\xi}{dt} = (\lambda_p - \xi)\gamma$$

is used in results for the correlation between change and initial status. The growth curve implied by $d\xi/dt$ is a special case of (4) with the restriction that $\gamma_p = \gamma$ for all individuals:

$$\xi_p(t) = \lambda_p - (\lambda_p - \xi_p(0))e^{-\gamma t}. \tag{14}$$

The asymptote λ_p may differ over individuals. (Bryk, 1977, investigates the properties of a different restricted form of (4) in which $\lambda_p = 1$ for all individuals.)

In the analysis of properties of a collection of straight-line growth curves, t° served as a useful centering point for the time scale. For a collection of growth curves defined by (14) $\sigma^2_{\xi(t)}$ may be written:

$$\sigma^2_{\xi(t)} = \sigma^2_{\xi(t^\circ)} + (\sigma^2_\lambda - \sigma^2_{\xi(t^\circ)})(1 - e^{-\gamma(t - t^\circ)})^2,$$

where t°, the time at which the variance is a minimum, is defined by

$$t - t^\circ = \left(\frac{1}{\gamma}\right) \ln\left[\frac{\sigma_{\lambda[\lambda - \xi(t)]}}{\sigma^2_{[\lambda - \xi(t)]}}\right]. \tag{15}$$

The t° defined in (15) is a different quantity from the t° defined for straight-line growth.

Correlation of change and initial status. The consequence of the choice of t_I for the correlation $\rho_{\xi(t_I)\Delta}$ (where $\Delta = \Delta(t_I, t_I + \tau)$) is seen from $\rho_{\xi(t)\Delta}$ as a function of t:

$$\rho_{\xi(t)\Delta} = \frac{1 - e^{-\gamma(t-t^\circ)}}{\left(\dfrac{\sigma^2_{\xi(t^\circ)}}{\sigma^2_\lambda - \sigma^2_{\xi(t^\circ)}} + (1 - e^{-\gamma(t-t^\circ)})^2\right)^{1/2}}, \tag{16}$$

where t° is defined in (15). Figure 5 displays $\rho_{\xi(t)\Delta}$ as a function of t. As with the plot in Figure 1, Figure 5 shows that $\rho_{\xi(t)\Delta}$ is a monotonically increasing function of t. Only at $t = t^\circ$ is $\rho_{\xi(t)\Delta} = 0$. Also, *$\xi(t^\circ)$ is uncorrelated with $d\xi/dt$ at any t. The function $\rho_{\xi(t)\Delta}$ has* maximum gradient at $t = t^\circ$.

The function $\rho_{\xi(t)\Delta}$ has a lower asymptote of -1 and an upper asymptote of $[1 - \rho^2_{\lambda\xi(t^\circ)}]^{1/2}$. As $\rho^2_{\lambda\xi(t^\circ)}$ (which equals $\sigma^2_{\xi(t^\circ)}/\sigma^2_\lambda$) approaches one, the upper asymptote of $\rho_{\xi(t)\Delta}$ approaches zero. When the condition for a finite t° (that there exists a t such that $\sigma^2_{\xi(t)} < \sigma^2_\lambda$) is not satisfied, $\rho_{\xi(t)\Delta}$ will always be negative. Only relatively uninteresting collections of growth curves lack a finite t° which would require $\rho_{\xi(t)\xi(t+\tau)}$ near one for all t.

As with straight-line growth, the Law of Initial Values is satisfied for $t_I < t^\circ$; moreover, if t° is not finite, then LIV always pertains. The two alternative statements of the Overlap Hypothesis—$\rho_{\xi(t_I)\xi(t_I+\tau)} = \sigma_{\xi(t_I)}/\sigma_{\xi(t_I+\tau)}$ or $\rho_{\xi(t_I)\Delta} = 0$—are satisfied only for $t_I = t^\circ$. And, in accordance with "fan spread" $\sigma^2_{\xi(t+\tau)} > \sigma^2_{\xi(t)}$ if and only if $t > t^\circ - (1/\gamma) \cdot \ln[2/(1 + e^{-\gamma\tau})]$. For example, Preece (1982) employs the growth curve in (14) as "a plausible growth function for a cognitive variable" (p. 761) and describes "droopy fan-spread growth" as collections of growth curves having $\sigma^2_{\xi(0)} = 0$.

Regression of change on initial status. The regression coefficient $\beta_{\Delta\xi(t_I)}$ also depends crucially on the t chosen to be t_I. The expression for $\beta_{\Delta\xi(t)}$ as a function of t demonstrates this dependence:

$$\beta_{\Delta\xi(t)} = \frac{(1 - e^{-\gamma\tau})e^{-\gamma(t-t^\circ)}(1 - e^{-\gamma(t-t^\circ)})}{\left(\dfrac{\sigma^2_{\xi(t^\circ)}}{\sigma^2_\lambda - \sigma^2_{\xi(t^\circ)}}\right) + (1 - e^{-\gamma(t-t^\circ)})^2}. \tag{17}$$

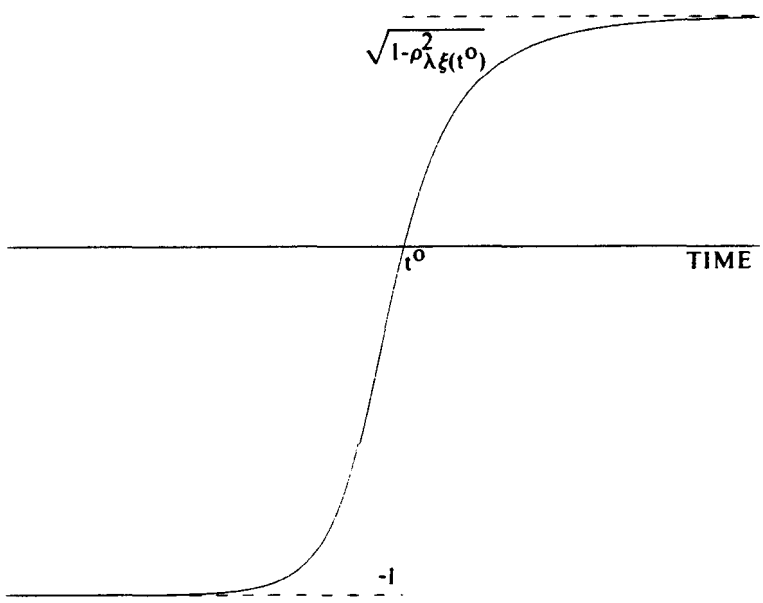

FIGURE 5
Plot of $\rho_{\xi(t)\Delta}$ against time for the restricted ($\gamma_p = \gamma$) exponential growth model. The values of the asymptotes are indicated at the dashed lines.

The function $\beta_{\Delta\xi(t)}$ has a maximum value of $(1 - e^{-\gamma t})[1 - \rho_{\lambda\xi(t^\circ)}]/2\rho_{\lambda\xi(t^\circ)}$ at $t = t^\circ + (1/\gamma) \ln[1 + \rho_{\lambda\xi(t^\circ)}]$ and a minimum value of $-(1 - e^{-\gamma t})(1 + \rho_{\lambda\xi(t^\circ)})/2\rho_{\lambda\xi(t^\circ)}$ at $t = t^\circ + (1/\gamma) \cdot \ln(1 - \rho_{\lambda\xi(t^\circ)})$. The lower asymptote of $\beta_{\Delta\xi(t)}$ is $-(1 - e^{-\gamma t})$ and the upper asymptote is zero.

The quantities $\rho_{\xi(t)\Delta}$ and $\beta_{\Delta\xi(t)}$ would have far more complex forms than those in (16) and (17) for a general model in which γ_p differed over individuals. With the restriction $\gamma_p = \gamma$, the methods of Rao (1958) indicate a transformation of time to $1 - e^{-\gamma t}$ for defining a "metameter" to linearize these growth curves. This transformation allows the development for this restricted exponential growth to be simplified; under the transformation indicated by the metameter, results for restricted exponential growth are analogous to results for straight-line growth.

2.4 Selected Empirical Studies

The previous parts of this section have presented mathematical properties of initial status as a correlate of change for both constant and non-constant rate of change models of individual growth. In this section selected empirical studies of the relation between change and initial status are reviewed. These studies provide an opportunity to illustrate the applicability of our results to empirical research.

In the empirical research literature a variety of approaches have been used to assess the relation between change and initial status. The simplest, and least attractive, approach is the use of the observed-score correlation, $r_{X_1(X_2 - X_1)}$ to estimate $\rho_{\xi(t_1)\Delta(t_1, t_2)}$. Although the severe negative bias of the observed score correlation was first reported more than fifty years ago, empirical assessments of the Overlap Hypothesis (Roff, 1941) and the Law of Initial Values (Campbell, 1981; Wilder, 1957), for example, have been based on $r_{X_1(X_2 - X_1)}$. Second, improved estimators of $\rho_{\xi(t_1)\Delta}$ are used to correct by disattenuation, or to otherwise eliminate, the bias in the observed score correlation (Messick, 1981; Thompson, 1924; Thorndike, 1966). A third approach employs the correlation structure generated by a specified growth model (e.g., the simplex correlation structure used in Werts & Hilton, 1977). A fourth approach, which corresponds most closely to the formulation of this paper, estimates the relation between change and initial status by modeling both individual growth and individual differences in growth (Blomqvist, 1977).

Thorndike (1966). Thorndike analyzed data from the Harvard Growth Study consisting of 6 yearly assessments of intelligence at ages 9 through 14 for 593 individuals. The existence of two measures of intelligence at each t_i ($i = 9, 10, 11, 12, 13, 14$) allowed Thorndike to compute correlations of the form $r_{X_i'(X_{i+k} - X_i)}$ (where X_i and X_i' denote the different measures at time t_i). This correlation avoids the negative bias due to the common error component of X_i and $X_{i+k} - X_i$ (Messick, 1981, Equations 6 & 7) and is subject only to attenuation from the unreliability of the measures X_i' and $X_{i+k} - X_i$. Thorndike's analysis of the six waves of data averages the $r_{X_i'(X_{i+k} - X_i)}$ over i for each time interval of length k, and positive correlations between change and initial status are found for each k (Thorndike, Table 1). Unfortunately, this analysis sacrifices considerable information about intellectual growth by breaking up the six waves of data into assorted two-wave pieces. Moreover, the dependence of the correlation on the time of initial status argues against the averaging of these correlations because, for example, the parameter estimated by $r_{X_9'(X_{10} - X_9)}$ may differ considerably from the parameter estimated by $r_{X_{13}'(X_{14} - X_{13})}$.

Werts and Hilton (1977). The ETS Growth Study provides repeated measurements of a number of academic abilities at 4 points in time (Grades 5, 7, 9 and 11) for nearly 2500 students. For each of 9 measures of achievement, Werts and Hilton estimated a correlation between true change and true initial status. Each estimate was obtained from the fit of a simplex correlation structure using LISREL (Werts & Hilton, Equations 2 & 4). (The modeling of the covariance or correlation structure does not make full use of the

longitudinal data and could be described as studying "growth" by ignoring individual change over time.) This approach yields only an estimate of $\rho_{\xi(t_7)\Delta(t_7, t_9)}$ from the four waves of data. The estimated correlation is positive for some measures and negative for others (their Table 3). However, Werts and Hilton's "status-gain" correlation with Grade 7 as time of initial status cannot be interpreted as applying to other choices of time of initial status. A related analysis by Werts, Linn and Jöreskog (1977), using measures of achievement on 8 occasions, averages 5 estimates of $\rho_{\xi(t_i)\Delta(t_i, t_{i+1})}$ (where $i = 2, \ldots, 6$) to produce a "unique" correlation of .39.

Blomqvist (1977). Blomqvist (1977) obtains a maximum likelihood estimator for $\beta_{\Delta\xi(t_I)}$ using straight-line growth (Blomqvist, Equation 3.2) and a representation of individual differences in growth as a function of initial status (Blomqvist, Equation 3.1), which in our notation is

$$E(\theta \mid \xi(0)) = \mu_\theta + \beta_{\theta\xi(0)}(\xi(0) - \mu_{\xi(0)}).$$

For $t_I = 0$, Blomqvist obtains maximum likelihood estimates of the elements of the covariance matrix of $\xi(0)$ and θ which yields his estimate of the regression of change on initial status (and illustrated with longitudinal data on systolic blood pressure). Our results for initial status and straight-line growth allow the adaptation of Blomqvist's estimation procedure to provide maximum likelihood estimates of t° and of $\beta_{\Delta\xi(t_I)}$ and $\rho_{\Delta\xi(t_I)}$.

2.5 *Regression Toward the Mean*

The relation between change and initial status is central to understanding "regression toward the mean" (also called the "regression effect"). The few formal statements of regression toward the mean in the literature define the regression toward the mean in standard deviation units; for example, Furby (1973, p. 174) states regression toward the mean for a time 1 measure x and a time 2 measure y as: "for a given score on x (e.g., x'), the corresponding mean score on y (e.g., y') is closer to $\bar{Y}$ in standard deviation units than x' is to $\bar{X}$ in standard deviation units." See also Nesselroade et al. (1980, p. 623). Thus, in the population, regression toward the mean for true scores at times t_1 and t_2 is said to occur when

$$\frac{E[\xi(t_2) \mid \xi(t_1) = C] - \mu_{\xi(t_2)}}{\sigma_{\xi(t_2)}} < \frac{C - \mu_{\xi(t_1)}}{\sigma_{\xi(t_1)}}. \tag{18}$$

Because this inequality is satisfied whenever $\rho_{\xi(t_1)\xi(t_2)} < 1$, regression toward the mean is thought to be omnipresent. We regard the formulation in (18) as a harmless mathematical tautology, and one which provides little insight for the study of change.

Following Rogosa et al. (1982, p. 735), it is far more useful to formulate the idea of regression toward the mean as

$$E[\xi(t_2) \mid \xi(t_1) = C] - \mu_{\xi(t_2)} < C - \mu_{\xi(t_1)}. \tag{19}$$

Only if $\sigma_{\xi(t_1)}$ and $\sigma_{\xi(t_2)}$ and constrained to be equal, as is done in Lord (1963, p. 21) and in Furby (1973, p. 173), is (19) equivalent to (18). The inequality in (19) better reflects an intuitive idea of regression toward the mean because in (19) "*closer*" is in terms of the actual metric of ξ. Healy and Goldstein (1978) also recognize the consequences of stating regression toward the mean in standard deviation units as opposed to the original metric.

Only when $\rho_{\xi(t_1)\Delta} < 0$ (where $\Delta = \Delta(t_1, t_2)$) is (19) satisfied. Previous results in this section indicate the conditions for $\rho_{\xi(t_1)\Delta} < 0$ for straight-line growth, the simplex model, and exponential growth. If $\xi(t_1)$ and $\xi(t_2)$ are constrained to have equal variance, then $\rho_{\xi(t_1)\Delta}$ must be negative unless $\rho_{\xi(t_1)\xi(t_2)} = 1$; consequently, regression toward the mean pertains. Actually, a separate literature on the regression effect is superfluous because

regression toward the mean (or lack thereof) is simply another manifestation of the correlation between change and initial status.

Severity of the regression effect. The degree to which the regression effect pertains has not previously been addressed. An obvious measure of the severity of the regression effect is the ratio:

$$\frac{E[\xi(t_2) \mid \xi(t_1) = C] - \mu_{\xi(t_2)}}{C - \mu_{\xi(t_1)}} = 1 + \beta_{\Delta\xi(t_1)}.$$

Equations (11), (12), and (17) describe $\beta_{\Delta\xi(t_1)}$ for straight-line, simplex, and exponential growth models, respectively.

Regression effect in multiwave data. Nesselroade et al. (1980) extended previous analyses by investigating regression toward the mean over more than two points in time. Their findings are limited by the exclusive consideration of standardized variables, and, as a result, regression toward the mean is present between any two points in time in all of their "autocorrelation systems." The main growth models used by Nesselroade et al. specify a simplex model for $\xi(t_i)$, with the analysis of regression toward the mean focusing on the correlation matrix of the X_i ($i = 1, \ldots, T$). Their "decreasing autocorrelation" model specifies $B_i = B < 1$, and their "accumulating advantage" model specifies $B_i = [1 + (i - 1)a]$ with $a > 0$. For the "accumulating advantage" model, Nesselroade et al. use the term "egression from the mean" to describe a regression toward the mean that is less severe between t_1 and t_3 than between t_1 and t_2 (even though there is regression toward the mean between t_1 and t_3). Perhaps a better use of this term would be egression from the mean as the opposite of regression to the mean, which would exist over the time interval $[t_i, t_{i+k}]$ if and only if $\rho_{\xi(t_i)\Delta(t_i, t_{i+k})} > 0$.

3. Reconsidering Conventional Procedures for Exogenous Characteristics

This section considers the properties of procedures commonly used in the behavioral sciences for assessing relations between change and exogenous individual characteristics. The specific procedures considered are: (a) correlations with the "base-free" measure of change from Tucker, Damarin and Messick (1966); (b) the partial correlation methods from Lord (1958, 1963); and (c) the regression model of Werts and Linn (1970). The technical results for this section are obtained using the representation for systematic individual differences in straight-line growth:

$$\begin{aligned} \xi_p(t) &= \xi_p(t^\circ) + \theta_p(t - t^\circ), \quad \text{and} \\ E(\theta \mid W) &= \mu_\theta + \beta_{\theta W}(W - \mu_W). \end{aligned} \tag{20}$$

Similar kinds of results could also be obtained using more complex models of individual growth and more complex relations (e.g., multiple W's) between the parameters and the exogenous characteristics.

3.1 Quantities from the Measurement of Change Literature

Table 2 displays the three quantities widely used in the behavioral sciences for the study of correlates or predictors of change. Each of these quantities incorporates information from only two time points, labeled t_1 and t_2. We denote $\xi(t_2) - \xi(t_1)$ by Δ and denote the Tucker, Damarin and Messick (1966) "base-free" measure of change by ϕ; $\phi = \xi(t_2) \cdot \xi(t_1)$, the residual from the $\xi(t_2)$ on $\xi(t_1)$ regression. In the Table, each quantity is identified with a "Source," and alternative expressions for each of these quantities are presented in the column labeled "Equivalences." Much attention has been given to these simple relations. Specifically, the ability to express each of the three quantities solely in terms of $\xi(t_1)$, $\xi(t_2)$ and W has lead many authors to the mistaken view that individual

TABLE 2

Summary of the Quantities Used in the Measurement of Change Literature in Assessing Correlates and Predictors of Change

Quantity	Source	Equivalences
$\rho_{\phi W}$	Tucker, Damarin & Messick (1966)	$\rho_{(\xi(t_2)\cdot\xi(t_1))W}$ $\rho_{(\Delta\cdot\xi(t_1))W}$
$\rho_{\Delta W\cdot\xi(t_1)}$	Lord (1958, 1963)	$\rho_{\xi(t_2)W\cdot\xi(t_1)}$ $\rho_{\phi W\cdot\xi(t_1)}$ $\rho_{\phi W}/\sqrt{1-\rho^2_{\xi(t_1)W}}$
$\beta_{\xi(t_2)W\cdot\xi(t_1)}$	Werts & Linn (1970)	$\beta_{\Delta W\cdot\xi(t_1)}$ $\beta_{\phi W\cdot\xi(t_1)}$ $\beta_{\phi W}/(1-\rho^2_{\xi(t_1)W})$

change has no role in the study of correlates and predictors of change (cf. Bohrnstedt, 1969, p. 128; Cronbach & Furby, 1970, pp. 79, 80; Kessler, 1977, p. 59; Linn & Slinde, 1977, p. 129; O'Connor, 1972, pp. 74, 76, 78, 95; Werts & Linn, 1970, p. 21).

Because the measurement of change literature differs markedly from the formulation of systematic individual differences in growth in this paper, it is useful to examine the rationale behind the quantities in Table 2. According to Tucker et al. (1966), the correlation between W and the "base-free' measure of change seeks to identify "which variables contribute to change in ways that are independent of initial position" (p. 466). Regarding the partial correlation, Lord (1963) argues: "It is not the total group correlation $\rho_{(X_2-X_1)W}$ but rather the partial correlation $\rho_{(X_2-X_1)W\cdot X_1}$ $(=\rho_{X_2W\cdot X_1})$ that is usually of greatest interest. A further complication arises at this point. It is really not $\rho_{(X_2-X_1)W\cdot X_1}$ that we want, but $\rho_{\Delta W\cdot\xi(t_1)}$" (p. 35, Lord's notation altered). The interpretation of the parameter $\beta_{\xi(t_2)W\cdot\xi(t_1)}$ provided by Werts and Linn (1970) is "that when the [true] initial status is an independent variable [in the prediction $\xi(t_2)$], the regression coefficient for the other independent variable [W] is the influence of that variable on growth" (p. 18, see also p. 21). This regression coefficient is also central in structural equation or LISREL models for longitudinal data (Wiley & Harnischfeger, 1973; Sörbom, 1976).

The three quantities have in common the attempt to control or statistically adjust for initial status, a strategy identified with residual change measures. (See also the "Equivalences" in Table 2.) Rogosa et al. (1982) in their analysis of the statistical and logical shortcomings of residual change procedures interpret the true residual change score as seeking to determine how much an individual would have changed on ξ if everyone had "started out equal." The logical difficulties of the adjustments for initial status are summarized by the questions, Equal on *what*? and, Started out *when*? These concerns are prominent in the following analyses of the properties of the quantities in Table 2.

3.2 *Technical Properties of Conventional Procedures*

The major misconception that dominates the use and interpretation of the quantities in Table 2 is that these quantities are thought to be "independent" of initial status. Although the residual change measure, $\xi(t_I + \tau) \cdot \xi(t_I)$, is uncorrelated (over persons) with $\xi(t_I)$, both the residual change measure and the quantities in Table 2 are very much dependent on the choice of the time at which initial status is defined. The results to follow demonstrate the fallacy of "controlling for" or "purging" initial status from measures of the association between change and W. The basic results show the dependence of the quantities in Table 2 on the choice of time of initial status.

3.2.1 *Properties of Correlational Measures*

The correlations $\rho_{[\xi(t_I+\tau)\cdot\xi(t_I)]W}$ (Tucker et al., 1966) and $\rho_{\xi(t_I+\tau)W\cdot\xi(t_I)}$ (Lord, 1963) have a dependence on the time of initial status determined by:

$$\rho_{[\xi(t+\tau)\cdot\xi(t)]W} = \frac{\kappa\rho_{W\theta} - (t - t^\circ)\rho_{W\xi(t^\circ)}}{(\kappa^2 + (t - t^\circ)^2)^{1/2}}, \quad \text{and} \tag{21}$$

$$\rho_{\xi(t+\tau)W\cdot\xi(t)} = \frac{\kappa\rho_{W\theta} - (t - t^\circ)\rho_{W\xi(t^\circ)}}{(\kappa^2 + (t - t^\circ)^2 - [\kappa\rho_{W\xi(t^\circ)} + (t - t^\circ)\rho_{W\theta}]^2)^{1/2}}. \tag{22}$$

Both $\rho_{[\xi(t_I+\tau)\cdot\xi(t_I)]W}$ and $\rho_{\xi(t_I+\tau)W\cdot\xi(t_I)}$ depend on the time of initial status $t = t_I$ and do not depend on, τ, the interval between "pretest" and "post test."

Inspection of the numerators in (21) and (22) reveals that both correlations are zero at the same value of t. This time is denoted by t^u, where

$$t^u = t^\circ + \kappa\left(\frac{\rho_{W\theta}}{\rho_{W\xi(t^\circ)}}\right).$$

When t^u exists (i.e., $\rho_{W\xi(t^\circ)} \neq 0$), (21) can be rewritten:

$$\rho_{[\xi(t+\tau)\cdot\xi(t)]W} = \frac{-\rho_{W\xi(t^\circ)}(t - t^u)}{(\kappa^2 + (t - t^\circ)^2)^{1/2}}. \tag{23}$$

Furthermore, by defining a time t^l such that

$$t^l = t^\circ - \kappa\left(\frac{\rho_{W\xi(t^\circ)}}{\rho_{W\theta}}\right),$$

(22) can be rewritten:

$$\rho_{\xi(t+\tau)W\cdot\xi(t)} = \frac{-\rho_{W\xi(t^\circ)}(t - t^u)}{(\kappa^2 + (t - t^\circ)^2 - \rho^2_{W\theta}(t - t^l)^2)^{1/2}}, \tag{24}$$

providing $\rho_{W\theta} \neq 0$ (i.e., t^l exists). At $t = t^l$ both correlations attain their maximum value of $[\rho^2_{W\xi(t^\circ)} + \rho^2_{W\theta}]^{1/2}$ (which is less than or equal to 1 because $\rho^2_{W\theta\cdot\xi(t^\circ)} \leq 1$ and $\rho_{\theta\xi(t^\circ)} = 0$). At $t = t^\circ$, $\rho_{[\xi(t+\tau)\cdot\xi(t)]W} = \rho_{W\theta}$ because $\xi(t^\circ + \tau)\cdot\xi(t^\circ) = \xi(t^\circ + \tau) - \xi(t^\circ) = \tau\theta$. Figure 6 displays, for $\rho_{W\theta} > 0$, $\rho_{[\xi(t+\tau)\cdot\xi(t)]W}$, from (23), and $\rho_{\xi(t+\tau)W\cdot\xi(t)}$ from (24). (In these and subsequent figures $\rho_{W\xi(t^\circ)} > 0$.) The correlation $\rho_{[\xi(t+\tau)\cdot\xi(t)]W}$ approaches asymptotes of $\mp\rho_{W\xi(t^\circ)}$ as t approaches $\pm\infty$. Also, $\rho_{\xi(t+\tau)W\cdot\xi(t)}$ approaches asymptotes of $\mp\rho_{W\xi(t^\circ)}/(1 - \rho^2_{W\theta})^{1/2}$ as t approaches $\pm\infty$.

The most salient feature of these mathematical functions for understanding systematic individual differences in growth is the strong dependence of the correlations on the time of initial status. The practical import of this dependence for empirical research on correlates of change is that an investigator using the methods of Lord (1963) or Tucker et al. (1966) may well be estimating a very different parameter depending on whether the

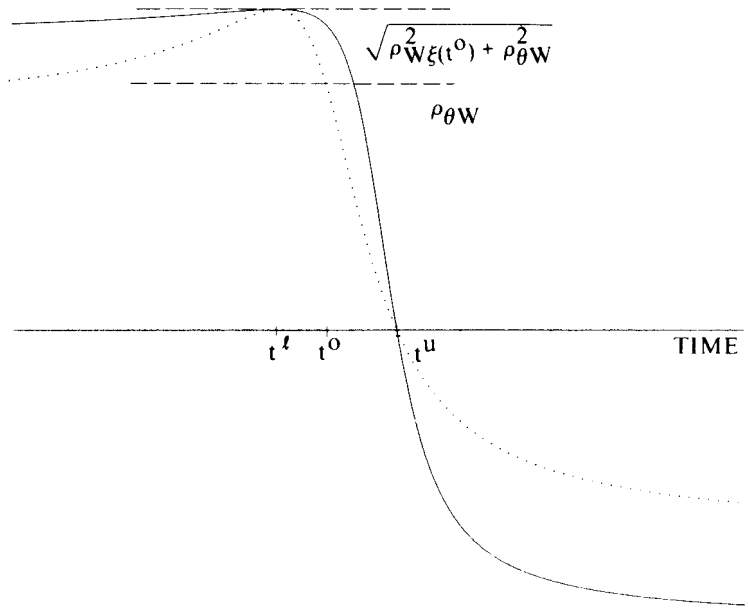

FIGURE 6
Plots of $\rho_{\xi(t+\tau)W\cdot\xi(t)}$ (solid line) and $\rho_{[\xi(t+\tau)\cdot\xi(t)]W}$ (dotted line) against time for straight-line growth. Values of the maximum and $\rho_{\theta W}$ are indicated at the dashed lines.

investigator, for example, begins a study of school learning when the students are in the third grade, the fifth grade, the seventh grade, and so forth. That is, for initial status defined at the beginning of third grade, an exogenous characteristic that the Lord or Tucker et al. approaches showed to be a strong correlate of change, might well appear to have no relationship with change if, instead, initial status were defined to be at the beginning of fifth grade. Furthermore, the dependence on time of initial status is most severe in the neighborhood of t° (see Figure 6), and t° appears to be reasonably near the time of initial status in much empirical research (judging from the frequent findings of small, negative correlations between observed change and observed initial status).

In addition to the ambiguity induced by the dependence of $\rho_{[\xi(t_I+\tau)\cdot\xi(t_I)]W}$ and $\rho_{\xi(t_I+\tau)W\cdot\xi(t_I)}$ on t_I both correlations may differ markedly from $\rho_{W\theta}$. To illustrate, two numerical examples based on the collection of growth curves shown in Figure 3 are presented.[2] The first example illustrates the properties of $\rho_{[\xi(t+\tau)\cdot\xi(t)]W}$ and $\rho_{\xi(t_I+\tau)W\cdot\xi(t_I)}$ when systematic individual differences in growth are not present ($\rho_{W\theta} = 0$). The second example incorporates strong systematic individual differences in growth ($\rho_{W\theta} = .7$).

Even when there are no systematic individual differences in growth, the quantities $\rho_{[\xi(t_I+\tau)\cdot\xi(t_I)]W}$ and $\rho_{\xi(t_I+\tau)W\cdot\xi(t_I)}$ may be large and positive or large and negative depending on the choice of t_I. Figure 7 illustrates that such distortions are consistent with plausible correlation structures. Also, Figure 7 shows that different choices of t_I (between 1 and 5) yield very different values of the correlations (e.g., compare the values of the correlations for initial status defined at time 1 with the values for initial status defined at time 4). For choices of t_I not near t° both correlations differ markedly from $\rho_{W\theta}$.

Mathematical results relevant to Figure 7 are found from (21) and (22) in the special

[2] As $\rho_{[\xi(t+\tau)\cdot\xi(t)]W}$ and $\rho_{\xi(t+\tau)W\cdot\xi(t)}$ are independent of τ it is convenient to think of $\tau = 1$ when linking the between-wave correlation matrix in Table 1 with quantities displayed in Figures 7 and 8.

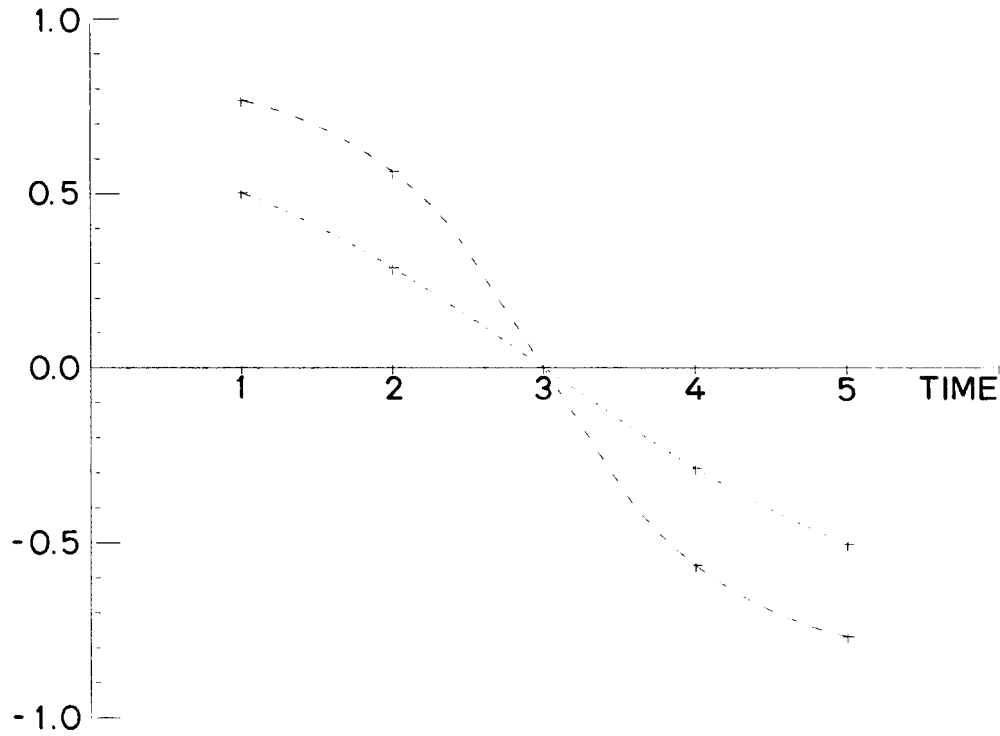

FIGURE 7
Values of $\rho_{\xi(t+\tau)W\cdot\xi(t)}$ (dashed line) and $\rho_{[\xi(t+\tau)\cdot\xi(t)]W}$ (dot-dash line) with $\rho_{W\theta} = 0$ for the collection of growth curves in Figure 3.

case of $\rho_{W\theta} = 0$:

$$\rho_{[\xi(t+\tau)\cdot\xi(t)]W} = \frac{-(t-t^{\circ})\rho_{W\xi(t^{\circ})}}{(\kappa^2 + (t-t^{\circ})^2)^{1/2}},$$

$$\rho_{\xi(t+\tau)W\cdot\xi(t)} = \frac{-(t-t^{\circ})\rho_{W\xi(t^{\circ})}}{(\kappa^2 + (t-t^{\circ})^2 - \kappa^2\rho_{W\xi(t^{\circ})})^{1/2}}.$$

Both correlations have asymptotes of $\mp\rho_{W\xi(t^{\circ})}$ for t approaching $\pm\infty$. In the example in Figure 7, $\rho_{W\xi(t^{\circ})} = .91$.

In contrast to the previous example, Figure 8 displays values of the two correlations with the specification of strong systematic individual differences in growth; $\rho_{W\theta} = .7$. Using the same collection of growth curves from Figure 3, values of the correlations are displayed for times of initial status between 3 and 8; $t^{u} = 6.5$ and $\rho_{W\xi(t^{\circ})} = .6$. The most notable feature of this example is that these correlations indicate that "change" and W are negatively associated when, in fact, a strong positive association exists ($\rho_{W\theta} = .7$)!

Properties of $\rho_{\xi(t)W}$. The correlation $\rho_{\xi(t)W}$ has the role in the study of correlates of change of the "true dependent change" correlation in Tucker et al. (1966). This correlation indicates systematic individual differences in the magnitude of ξ at time t. Tucker et al. suggest that estimates of $\rho_{\xi(t_I)W}$ be used to "learn which variables determine the subject's position on the initial base-line" (p. 466). In addition, $\rho_{\xi(t)W}$ links $\rho_{[\xi(t+\tau)\cdot\xi(t)]W}$ and $\rho_{\xi(t+\tau)W\cdot\xi(t)}$ as shown in the "Equivalence" column of Table 2. For straight-line growth, a general form for $\rho_{\xi(t)W}$ is

$$\rho_{\xi(t)W} = \frac{\kappa\rho_{W\xi(t^{\circ})} + (t-t^{\circ})\rho_{W\theta}}{(\kappa^2 + (t-t^{\circ})^2)^{1/2}}.$$

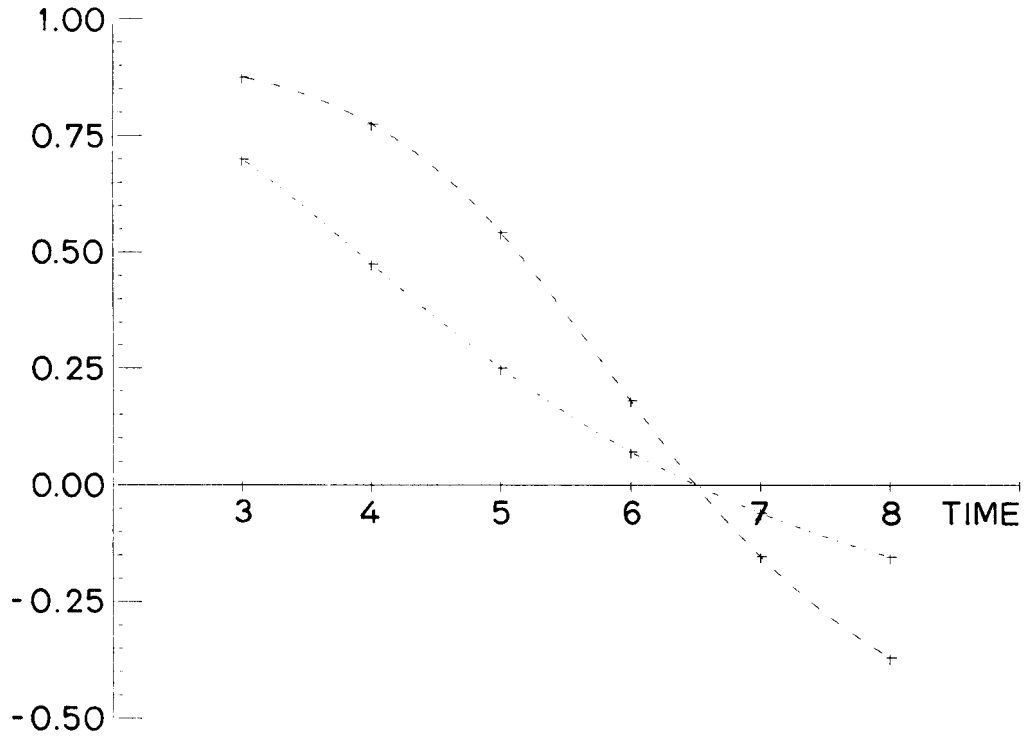

FIGURE 8
Values of $\rho_{\xi(t+\tau)W\cdot\xi(t)}$ (dashed line) and $\rho_{[\xi(t+\tau)\cdot\xi(t)]W}$ (dot-dash line) with $\rho_{W\theta} = .7$ for the collection of growth curves in Figure 3.

Providing $\rho_{W\theta} \neq 0$ (i.e., t^l exists):

$$\rho_{\xi(t)W} = \frac{\rho_{W\theta}(t - t^l)}{(\kappa^2 + (t - t^\circ)^2)^{1/2}}.$$

The correlation $\rho_{\xi(t)W}$ achieves a maximum value of $[\rho^2_{W\theta} + \rho^2_{W\xi(t^\circ)}]^{1/2}$ at $t = t^u$, equals zero at $t = t^l$, and has asymptotes of $\pm\rho_{W\theta}$.

3.2.2 *Measures for Predictors of Change*

The properties of the regression parameter $\beta_{\xi(t_I+\tau)W\cdot\xi(t_I)}$ used by Werts and Linn (1970) to represent the influence of W on growth are similar to those of the correlations. For systematic individual differences in straight-line growth, $\beta_{\xi(t+\tau)W\cdot\xi(t)}$ has the form:

$$\beta_{\xi(t+\tau)W\cdot\xi(t)} = \frac{\tau[\kappa\rho_{W\theta} - (t - t^\circ)\rho_{W\xi(t^\circ)}][\sigma_{\xi(t^\circ)}/\sigma_W]}{\kappa^2 + (t - t^\circ)^2 - [\kappa\rho_{W\xi(t^\circ)} + (t - t^\circ)\rho_{W\theta}]^2}. \tag{25}$$

As $\beta_{\xi(t+\tau)W\cdot\xi(t)}$ is in the metric of ξ, it is proportional to τ, the time interval between "pretest" and "post test." Providing that t^u and t^l exist, (25) can be rewritten

$$\beta_{\xi(t+\tau)W\cdot\xi(t)} = \frac{-\tau(t - t^u)\rho_{W\xi(t^\circ)}[\sigma_{\xi(t^\circ)}/\sigma_W]}{\kappa^2 + (t - t^\circ)^2 - \rho^2_{W\theta}(t - t^l)^2}. \tag{26}$$

The regression coefficient is zero at $t = t^u$ and also approaches zero as t approaches $\pm\infty$. At $t = t^l$, $\beta_{\xi(t+\tau)W\cdot\xi(t)}$ equals $\tau\beta_{\theta W}$. A plot of $\beta_{\xi(t+\tau)W\cdot\xi(t)}$ as a function of t is shown in Figure 9. The regression coefficient has a minimum and a maximum at the times

$$t^u \pm \left(\frac{\sigma_{\xi(t^u)}}{\sigma_\theta}\right)\left(\frac{1 - \rho^2_{W\xi(t^u)}}{1 - \rho^2_{W\theta}}\right)^{1/2},$$

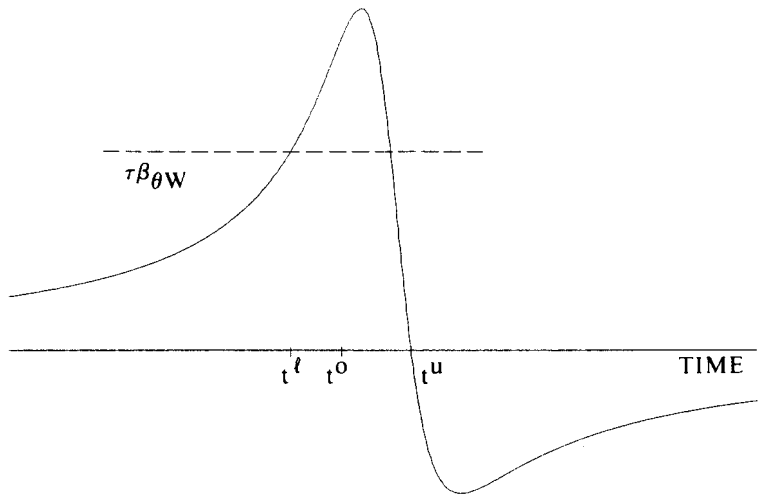

FIGURE 9
Plot of $\beta_{\xi(t+\tau)W\cdot\xi(t)}$ against time for straight-line growth. The value of $\tau\beta_{\theta W}$ is indicated at the dashed line.

respectively. As with the correlational measures, the most salient property of $\beta_{\xi(t_I+\tau)W\cdot\xi(t_I)}$ is the dependence on time of initial status.

For $\rho_{W\theta} = 0$, (25) becomes

$$\beta_{\xi(t+\tau)W\cdot\xi(t)} = \frac{-\tau(t-t^{\circ})\rho_{W\xi(t^{\circ})}[\sigma_{\xi(t^{\circ})}/\sigma_W]}{\kappa^2(1-\rho^2_{W\xi(t^{\circ})}) + (t-t^{\circ})^2}. \tag{27}$$

The function in (27) equals zero for $t = t^{\circ}$ and has minimum and maximum at $t^{\circ} \pm \kappa[1 - \rho^2_{W\xi(t^{\circ})}]^{1/2}$, respectively. Despite the specification that $\beta_{\theta W} = 0$, $\beta_{\xi(t_I+\tau)W\cdot\xi(t_I)}$ will be positive or negative for t_I less than or greater than t°.

Exponential growth. Under the model for exogenous change of Coleman (1968, Equation 11.15), which is rewritten in our (5), the dependence of the regression coefficient on the choice of t_I disappears. Recall that Coleman's model is a special case of systematic individual differences in exponential growth with the restrictions $\gamma_p = \gamma$ and, especially, $\rho_{\lambda W} = 1$. Under these restrictions (which cannot be expected to hold in practice), $\beta_{\xi(t+\tau)W\cdot\xi(t)} = \beta_{\lambda W}(1 - e^{-\gamma\tau})$.

Discussion

At least four purposes for studying change are prominent in the behavioral sciences: (a) the assessment of individual change, (b) the detection of correlates or predictors of change, (c) the comparison of change among experimental groups, and (d) the comparison of change among nonequivalent groups in quasiexperiments (see also Cronbach & Furby, 1970, pp. 77–80). Individual change was the focus of Rogosa et al. (1982), and the present paper moves on to correlates of change. "Understanding Correlates of Change" means how to think about and explicitly formulate systematic individual differences in growth. This understanding (which is achieved by "Modeling Individual Differences in Growth") is a necessary first step in the development of statistical methods to guide the design and analysis of empirical research. A major consequence of this understanding is a call to abandon the teachings of the "Avoid Change at Any Cost" School of Longitudinal Research which have dominated the measurement of change literature. This paper demonstrates that explicit consideration of change—through the parameters of a model for individual growth—is absolutely essential for any serious treatment of correlates of change.

This paper adopts a number of assumptions and restrictions of scope in order to make the presentation manageable. The models for individual growth that are employed are relatively simple, especially when compared with deterministic and stochastic growth models used in biological research or even with some psychological learning models (Gulliksen, 1934; Lewis, 1960). Moreover, these statistical models of individual growth do presume an ability to speak seriously of quantitative change in the variable under study; such a presumption is certainly not universally valid in behavioral research. Also, except for the special case of initial status as a correlate of change, only exogenous individual characteristics that are unchanging over time are included in the formulation of systematic individual differences in growth. Finally, the development of statistical methods for the estimation of systematic individual differences in growth lies outside the realm of this paper, although the paper does contribute to statistical estimation by identifying the key statistical parameters for the study of correlates of change.

The understanding of correlates of change is built up in the three main sections which make up this paper. In answering the question "What is a correlate of change?" the first section introduces a number of models for individual growth and specifies the dependence of the parameters of these growth models on exogenous individual characteristics. In addition to introducing the basic models used in the second and third sections, the first section seeks to emphasize the importance of the model for individual growth. In particular, an important distinction is drawn between analysis of individual differences in the amount of change over a specific time interval and analysis of individual differences in the parameters of a particular model for individual growth. Although analyzing the amount of change is conceptually a step in the right direction (as opposed to ignoring individual change entirely), it is inferior to the examination of individual differences in the parameters of an individual growth model (except in the special case of the constant rate of change model where these are interchangeable). Consequently, the common longitudinal design which obtains just two observations on each individual (and which, at best, allows estimation of only the amount of change) is rarely adequate for the study of systematic individual differences in growth. The importance of obtaining observations on each individual at more than two time points cannot be overstated.

The message of the second section "Using Initial Status as a Correlate of Change" can be stated as a multiple-choice item:

The correlation between change and initial status is

a) negative
b) zero
c) positive
d) all of the above.

Choice (d) is correct because the results of this section clearly demonstrate that there is no such thing as *the* correlation between change and initial status, although the determination of a unique correlation seems to have been the goal of much empirical research. Equations (9), (13), and (16) and Figures 1, 4, and 5 illustrate that the value of the correlation between change and initial status depends crucially upon the time at which initial status is defined. The dependence of the correlation on the time of initial status is determined by the form of the individual growth curves and by the individual differences in the parameters of the collection of individual growth curves. The same general conclusions apply to the regression of change on initial status; see (11), (12), and (17).

The third section "Reconsidering Conventional Approaches for Exogenous Characteristics" examines the properties of the correlational and regression procedures used in the measurement of change literature when the correlate is an exogenous individual characteristic. These procedures have in common the use of only two measurements on

each individual and the attempt to "purge initial status" from the relation between change and the exogenous characteristic. As the second section has demonstrated that the correlation between change and initial status is an ephemeral quantity (in the sense of depending on the choice of time of initial status), it is not surprising that serious deficiencies are found in these procedures. The mathematical results (using straight-line individual growth) demonstrate serious problems with each of these procedures (see, for example, Figures 6–9). The failure of these traditional approaches can be attributed to the absence of models for individual growth—that is, the attempt to analyze "change" while ignoring individual growth—and to the meager amount of information on individual change that can be obtained from only two measurements on each individual. Much closer to the understanding that this paper attempts to foster are the approaches to individual differences in learning that were present in the work of Woodrow (1940, 1946) and perhaps last seen in Stake's (1961) Psychometric Monograph.

References

Anderson, J. E. (1939). The limitations of infant and preschool tests in the measurement of intelligence. *Journal of Psychology, 8*, 351–379.

Anderson, T. W. (1960). Some stochastic process models for intelligence test scores. In K. J. Arrow, S. Karlin, & P. Suppes (Eds.), *Mathematical methods in the social sciences*. Stanford, CA.: Stanford University Press.

Bayley, N. (1949). Consistency and variability in the growth of intelligence from birth to eighteen years. *Journal of Genetic Psychology, 75*, 165–196.

Bayley, N. (1956). Individual patterns of development. *Child Development, 27*, 45–74.

Bayley, N. (1968). Behavioral correlates of mental growth: Birth to thirty-six years. *American Psychologist, 23*, 1–17.

Berliner, D. C. (1976). Impediments to the study of teacher effectiveness. *Journal of Teacher Education, 27*, 5–13.

Blomqvist, N. (1977). On the relation between change and initial value. *Journal of the American Statistical Association, 72*, 746–749.

Bloom, B. S. (1964). *Stability and change in human characteristics*. New York: Wiley.

Bohrnstedt, G. W. (1969). Observations on the measurement of change. In E. F. Borgatta (Ed.), *Sociological methodology 1969*. San Francisco: Jossey-Bass.

Borich, G. D. (1979). Implications for developing teacher competencies from process-product research. *Journal of Teacher Education, 30*, 77–86.

Bryk, A. S. (1977). An investigation of the effectiveness of alternative adjustment strategies in the analysis of quasi-experimental growth data. Unpublished doctoral dissertation, Harvard University, Graduate School of Education, Cambridge.

Bryk, A. S., & Weisberg, H. I. (1977). Use of the nonequivalent control group design when subjects are growing. *Psychological Bulletin, 84*, 950–962.

Campbell, M. E. (1981). Statistical procedures with the law of initial values. *Journal of Psychology, 108*, 85–101.

Coleman, J. S. (1968). The mathematical study of change. In H. M. Blalock & A. B. Blalock (Eds.), *Methodology in social research*. New York: McGraw-Hill.

Cook, T. D., & Campbell, D. T. (1979). *Quasi-experimentation: Design and analysis issues for field settings*. Skokie, IL: Rand McNally.

Cronbach, L. J., & Furby, L. (1970). How should we measure "change"—or should we? *Psychological Bulletin, 74*, 68–80.

Cronbach, L. J., & Snow, R. E. (1977). *Aptitudes and instructional methods: A handbook for research on interactions*. New York: Irvington.

Day, N. E. (1966). Fitting curves to longitudinal data. *Biometrics, 22*, 276–291.

Fisk, P. R. (1967). Models of the second kind in regression analysis. *Journal of the Royal Statistical Society (B), 29*, 266–281.

Furby, L. (1973). Interpreting regression toward the mean in developmental research. *Developmental Psychology, 8*, 172–179.

Garside, R. F. (1956). The regression of gains upon initial scores. *Psychometrika, 21*, 67–77.

Goldstein, H. (1979). *The design and analysis of longitudinal studies*. London: Academic Press.

Guire, K. E., & Kowalski, C. J. (1979). Mathematical description and representation of developmental change functions on the intra- and inter-individual levels. In J. R. Nesselroade & P. B. Baltes (Eds.), *Longitudinal research in the study of behavior and development*. New York: Academic Press.

Gulliksen, H. (1934). A rational equation of the learning curve based on Thorndike's law of effect. *Journal of General Psychology, 11*, 395–434.

Guttman, R., & Guttman, L. (1965). A new approach to the analysis of growth patterns: The simplex structure of intercorrelations of measurements. *Growth, 29*, 219–232.

Harnquist, K. (1968). Relative changes in intelligence from 13 to 18. *Scandinavian Journal of Psychology, 9*, 50–82.

Healy, M. J. R., & Goldstein, H. (1978). Regression to the mean. *Annals of Human Biology, 5*, 277–280.

Hicklin, W. J. (1976). A model for mastery learning based on dynamic equilibrium theory. *Journal of Mathematical Psychology, 13*, 79–88.

Hilgard, E. R. (1951). Methods and procedures in the study of learning. In S. S. Stevens (Ed.), *Handbook of Experimental Psychology*. New York: Wiley.

Hui, S. L., & Berger, J. O. (1983). Empirical Bayes estimation of rates in longitudinal studies. *Journal of the American Statistical Association, 78*, 753–760.

Hull, C. L. (1943). *Principles of behavior*. New York: Appleton-Century-Crofts.

Humphreys, L. G. (1960). Investigations of the simplex. *Psychometrika, 25*, 313–323.

Jöreskog, K. G. (1970). Estimation and testing of simplex models. *British Journal of Mathematical and Statistical Psychology, 23*, 121–145.

Keats, J. A. (1983). Ability measures and theories of cognitive development. In H. Wainer & S. Messick (Eds.), *Principles of modern psychological measurement: A festschrift for Frederick M. Lord*. Hillsdale, NJ: Lawrence Erlbaum.

Kessler, R. C. (1977). The use of change scores as criteria in longitudinal survey research. *Quality and Quantity, 11*, 43–66.

Kessler, R. C., & Greenberg, D. F. (1981), *Linear panel analysis*. New York: Academic Press.

Lacey, J. I. (1956). The evaluation of automatic responses: Toward a general solution. *Annals of the New York Academy of Sciences, 67*, 123–164.

Lacey, J. I., & Lacey, B. C. (1962). The law of initial value in the longitudinal study of autonomic constitution: Reproducibility of autonomic responses and response patterns over a four year interval. In W. M. Wolf (Ed.), Rythmic functions in the living system. *Annals of the New York Academy of Sciences, 98*, 1257–1290.

Lewis, D. (1960). *Quantitative methods in psychology*. New York: McGraw-Hill.

Linn, R. L., & Slinde, J. A. (1977). The determination of the significance of change between pre- and posttesting periods. *Review of Educational Research, 47*, 121–150.

Lord, F. M. (1958). Further problems in the measurement of growth. *Educational and Psychological Measurement, 18*, 437–454.

Lord, F. M. (1963). Elementary models for measuring change. In C. W. Harris (Ed.), *Problems in measuring change*. Madison, WI: University of Wisconsin Press.

McCall, R. B., Appelbaum, M. I., & Hogarty, P. S. (1973). Developmental changes in mental performance. *Monographs of the Society for Research in Child Development, 38* (No. 150).

Medley, D. M. (1979). The effectiveness of teachers. In P. L. Peterson & H. J. Walberg (Eds.), *Research on teaching: Concepts, findings and interpretations*. Berkeley: McCutchan.

Messick, S. (1981). Denoting the base-free measure of change. *Psychometrika, 46*, 215–217.

Nesselroade, J. R., Stigler, S. M., & Baltes, P. B. (1980). Regression toward the mean and the study of change. *Psychological Bulletin, 88*, 622–637.

Nielsen, F., & Rosenfeld, R. A. (1981). Substantive interpretations of differential equation models. *American Sociological Review, 46*, 159–174.

O'Connor, E. F. (1972). Extending classical test theory to the measurement of change. *Review of Educational Research, 42*, 73–98.

Preece, P. F. W. (1982). The fan-spread hypothesis and the adjustment for initial differences between groups in uncontrolled studies. *Educational and Psychological Measurement, 42*, 759–762.

Rao, C. R. (1958). Some statistical methods for comparison of growth curves. *Biometrics, 14*, 1–17.

Rao, C. R. (1965). The theory of least squares when the parameters are stochastic and its application to the analysis of growth curves. *Biometrics, 52*, 447–458.

Rees, A. H., & Palmer, F. H. (1970). Factors related to change in mental test performance. *Developmental Psychology Monographs, 3* (Part 2).

Richards, F. J. (1959). A flexible growth function for empirical use. *Journal of Experimental Botany, 10*, 290–300.

Roff, M. (1941). A statistical study of the development of intelligence test performance. *Journal of Psychology, 11*, 371–386.

Rogosa, D. R., Brandt, D., & Zimowski, M. (1982). A growth curve approach to the measurement of change. *Psychological Bulletin, 90*, 726–748.

Sagiv, A. (1979). General growth model for evaluation of an individual's progress in learning. *Journal of Educational Psychology, 71*, 866–881.

Salemi, M. K., & Tauchen, G. E. (1982). Estimation of nonlinear learning models. *Journal of the American Statistical Association, 77*, 725–731.

Sandland, R. L., & McGilchrist, C. A. (1979). Stochastic growth curve analysis. *Biometrics, 35*, 255–271.

Seigel, D. G. (1975). Several approaches for measuring average rates of change for a second degree polynomial. *The American Statistician, 29*, 36–37.

Soar, R. S. (1977). An integration of findings from four studies of teacher effectiveness. In G. D. Borich (Ed.), *The appraisal of teaching: Concepts and process.* Reading, MA: Addison-Wesley.

Soar, R. S. (1979). Teacher behavior related to pupil growth. In N. Bennett & D. McNamara (Eds.), *Focus on teaching.* New York: Longman.

Sörbom, D. (1976). A statistical model for the measurement of change. In D. N. M. de Gruijter & L. J. T. van der Kamp (Eds.), *Advances in psychological and educational measurement.* New York: Wiley.

Sørensen, A. B., & Hallinan, M. T. (1977). A reconceptualization of school effects. *Sociology of Education, 50*, 273–289.

Stake, R. E. (1961). Learning parameters, aptitudes and achievements. *Psychometric Monograph*, (No. 9).

Swamy, P. A. V. B. (1971). *Statistical inference in a random coefficient regression model.* New York: Springer-Verlag.

Swamy, P. A. V. B. (1974). Linear models with random coefficients. In P. Zarembka (Ed.), *Frontiers in econometrics.* New York: Academic Press.

Thomson, G. H. (1924). A formula to correct for the effect of errors of measuremnt on the correlation of initial values with gains. *Journal of Experimental Psychology, 7*, 321–324.

Thorndike, R. L. (1966). Intellectual status and intellectual growth. *Journal of Educational Psychology, 57*, 121–127.

Thurstone, L. L., & Ackerson, L. (1929). The mental growth curve for the Binet tests. *Journal of Educational Psychology, 20*, 569–583.

Tucker, L. R. (1979). Comment on a note on a base-free measure of change. *Psychometrika, 44*, 357.

Tucker, L. R., Damarin, F., & Messick, S. (1966). A base-free measure of change. *Psychometrika, 31*, 457–473.

Tuma, N. B., & Hannan, M. T. (1984). *Social dynamics: Models and Methods.* New York: Academic Press.

Veldman, D., & Brophy, J. (1974). Measuring teacher effects on pupil achievement. *Journal of Educational Psychology, 66*, 319–324.

Walberg, H. J., & Tsai, S. L. (1983). Matthew effects in education. *American Educational Research Journal, 20*, 359–373.

Werts, C. E., & Hilton, T. L. (1977). Intellectual status and intellectual growth, again. *American Educational Research Journal, 14*, 137–146.

Werts, C. E., & Linn, R. L. (1970). A general linear model for studying growth. *Psychological Bulletin, 73*, 17–22.

Werts, C. E., Linn, R. L., & Jöreskog, K. G. (1977). A simplex model for analyzing academic growth. *Educational and Psychological Measurement, 37*, 745–756.

Wiley, D. E., & Harnischfeger, A. (1973). Post hoc, ergo propter hoc: Problems in the attribution of change. *Studies of Educative Processes* (No. 7). Chicago: University of Chicago.

Wilder, J. (1957). The law of initial value in neurology and psychiatry. *Journal of Nervous and Mental Disease, 125*, 73–86.

Wilder, J. (1965). Pitfalls in the methodology of the law of initial value. *American Journal of Psychotherapy, 19*, 577–584.

Wishart, J. (1938). Growth rate determinations in nutrition studies with the bacon pig and their analysis. *Biometrika, 30*, 16–28.

Wohlwill, J. F. (1980). Cognitive development in childhood. In O. G. Brim, Jr. & J. Kagan (Eds.), *Constancy and change in human development.* Cambridge, MA: Harvard University Press.

Woodrow, H. (1940). Interrelations of measures of learning. *Journal of Psychology, 10*, 49–73.

Woodrow, H. (1946). The ability to learn. *Psychological Review, 53*, 147–158.

Manuscript received 12/6/83
Final version received 12/17/84